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ARBITRAGE AND MARTINGALE MEASURE

Synopsis of the Ph.D. Thesis

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Contents

1 Introduction

The most important problem of asset pricing is the mathematical characterization of the economically consistent models of financial markets. From the economic point of view the assumption of no-arbitrage – *i.e.* the assumption that one cannot do business in which one can only gain with strictly positive probability but without exposure to risk – seems a fairly mild condition and it is astonishing what far-reaching consequenes it has.

Under the Fundamental Theorem of Asset Pricing we mean such types of statements which assert an equivalence between the above-mentioned consistency condition and some mathematical properties of the stochastic process describing asset prices. The aim of the dissertation is to survey the literature and the mathematical and economic antecedents of the fundamental theorem of asset pricing for semimartingale models, to present a didactic unified treatment of the semimartingale approach of asset pricing and to simplify some of the original proofs. A new feature of the dissertation is that it does not focus on questions of pricing but on the relations of the arbitrage theory and the classical theory of economics.

The fundamental theorem of asset pricing, loosely speaking, asserts that the absence of arbitrage is equivalent to the existence of an equivalent probability measure, under which the discounted price process is a "martingale", which means that there is a new fictive probability under which one can't systematically gain on average. To be more precise, this statement in this form is only a principle which will become a verifiable statement if we specify the notions of "no-arbitrage" and "martingale" in it. We will see that the contents of these concepts may differ according to the types of the model.

No-arbitrage states that by using zero initial wealth there is only one payoff among the available net payoffs which is non negative, namely the constant zero. This means that no-arbitrage states that two sets intercept each-other only in one point. It is well-known that this situation can be described by the separating hyperplane theorem very well and in this context the martingale measure is determined by the coefficients of the separating hyperplane. To be able to use the separating hyperplane theorem we need two things. Firstly we must assure the sets to be convex and to be closed. The assumption of convexity is easy to guarantee. It is sufficient to suppose that any convex combinations of any set of admissible strategies are also admissible. Although the convexity of the strategic sets in general equilibrium theory is a widely used but controversial condition, in the theory of finance it is generally accepted. As will be seen, the problem concerns the closedness of the sets which must be separated, and the assurance of the closedness is the main difficulty of the theorems presented in the following.

The above separating hyperplane method can be applicable to the case of multidimensional models, as far as the problem remains finite dimensional. In this case the sets of net payoffs available by means of zero initial wealth – set K in the following – is trivially closed since a finite dimensional subspace of an arbitrary topological vectorspace is closed. In the relevant stochastic models, however, the space of contingent claims is usually infinite dimensional, so in this case – as we know from functional analysis – the problems are topological in nature. In this case the closedness of K will not be satisfied automatically, which is the main difficulty of the proof.

The usual textbook finite dimensional proof of the fundamental theorem demonstrates the essence of arbitrage pricing but it is misleading in several respects. A special feature of these models is the assumption that time periods are finite. If the numbers of trading periods is finite, and if we use unbounded resources we can reach positive payoff with probability one. Because these types of trading strategies are unrealistic, we must exclude them. As a result, in general, set K will not necessary be a subspace, but a cone. Another pleasant property of the finite dimensional model is that the primal space – using the terminology of general equilibrium, the commodity space, or in our case the space of contingent claims – and the dual space – that is the space containing the price system and the martingale measure – are depictable in the same coordinate system. More generally, an arbitrary separated topological vector space of n-dimension is isomorfic to the space of $\mathbb{R}^n$, so in this case there is no point in introducing the abstract notion of topology and making distinctions between primal and dual spaces. However, in topological vector spaces of infinite dimension the choice of the topology is not unique, so it is difficult to ensure both the primal space and its dual to fit the economical problem. Usually this does not hold either in the theory of general equilibrium or in the theory of arbitrage. In this case both in the theory of general equilibrium and the theory of arbitrage the existence of the adequate price system can be assured by restrictions on the preferencies of the agents.

This statement was first proved by M. Harrison and S. R. Pliska [24] for the case of finitely generated probability spaces. R. C. Dalang, A. Morton and W. Willinger [9] generalized the theorem for the case of general probability spaces and discrete finite time horizons. There is an equally important but less-known theorem by Kreps which is more relevant from the point of view of this dissertation. Kreps' theorem states that the condition of No Free Lunch is equivalent to the existence of an equivalent martigale measure.

Although this result can be applied to semimartingale models, the notion of no free lunch lacks clear economic interpretation. F. Delbaen and W. Schachermayer ([10], [11]) gave a satisfactory solution to the problem for semimartingale models. This latter result is one of the peak achievements of mathematical finance. Its proof is rather lengthy and uses deep results of functional analysis and the general theory of stochastic processes developed by the French Probability School, mainly by P. A. Meyer from the 60s on. Although the proof of the Delbaen-Schachermayer theorem was slightly simplified by Kabanov [30], its proof continues to be inaccessible for economists. It is less-known that there is a variant of the fundamental theorem the economic content and level of abstraction of which is similar to the Delbaen–Schachermayer theorem, but the proof of which demands a much more simple mathematical apparatus. Frittelli's fundamental theorem (:[20] and [21]) uses a new arbitrage concept that is equivalent to Kreps' no free lunch, but in contrast to the notion of no free lunch it is easy to interpret economically.

2 The Structure, Results and Methods of the Disertation

The dissertation focuses on semimartingale models, so the discrete time models and examples serve primarily to provide a simplified introduction. The tratment of these models is not so rigorous as that of the semimartingale models because exhaustive treatments of these models are available in the cited textbooks and monografies. The only exception is section 5.1. about the dual approach of portfolio optimisation which highlights the duality methods of convex analysis, namely the weak and strong theorems of Lagrangian duality, so it is different from the original, more elementary treatment of the subject matter in [12]. In the chapters on semimartingale models, however, I strive to present the most precise exposition possible, and in this summary I will focus on the content of these chapters. The mathematical apparatus in these chapters includes topology, measure theory, the duality theory of functional analysis, a bit of the theory of Hilbert spaces and set theory, some relatively new chapters on the general theory of stochastic processes and some results of convex analysis and the theory of Orlicz spaces.

Finally, I would like to highlight that the dissertation deliberately deals neither with special problems of asset pricing, such as the variants of Black–Scholes and Cox–Ross– Rubinstein model nor with the models involving transaction costs. Although they are related to the subject matter of the dissertation, I also do not deal with the second fundamental theorem of asset pricing and the problem of completeness.

While the first chapter outlines the connections between martingale measures and the notion of no-arbitrage, the second chapter of the dissertation presents the main antecedents of the Delbaen–Schachermayer theorem in mathematics and economics. In this chapter I first derive the martingale measure from the Arrow–Debreu equilibrium prices, and give one of its possible economic interpretations in the context of the Radner equilibrium. It is known that martingale measures represent a pricing functional defined not just on the replicable payoffs but on the set of all the possible contingent claims, so in this chapter the fundamental theorem of asset pricing appears as the problem of the extension of the pricing functional.

In the second chapter I define the notion of arbitrage for two period models, and point out the fact that for general probability spaces the no-arbitrage condition is not already a sufficient condition for the existence of the extended pricing functional, so we need stronger conditions, such as the conditions of no free lunch and viability. We will see that the extension of the fundamental theorem to the case of infinite dimension is not trivial. Owing to the topological problems arising, the classical separating theorems of functional analysis are not applicable. As mentioned, in this case, contrary to finite dimensional cases, we must explicitly distinguish between the space and its dual. In this case the separating hyperspaces are represented by the elements of the dual space. Obviously, we will choose an L^p space as the space of contingent claims. In this case, however, we are bound to encounter the same problem as the theory of general equilibrium did in the 50s, 60s and in the 70s. The commodity space in the general equilibrium theory usually is some space L^p, and the price system is a continuous linear functional defined on it. If we want to apply the Hahn–Banach type theorems, every choice of p poses a serious problem. On the one hand, if $p < \infty$ (e.g. [15], [25] and [9]), the positive ortant of the space L^p has no internal points, so the Hahn-Banach theorem is not applicable. On the other hand, if $p = \infty$ then the price system is not necessarily representable as a scalar product because the dual of space L^∞ is larger than space L^1 (se *e.g.*: [29]). In most of the economic applications the commodity space is space L^∞. In the general equilibrium theory, if we want the price system to be in L^1, we must make restrictions on the consumers' preferences (see *e.g.*: [3] and [39]). However, it is not obvious what the no-arbitrage condition has to do with the investor's preferences. One of the aims of the dissertation is to clarify this connection.

Kreps [35] proves – under some separability conditions – that the condition of viability is equivalent to the notion of No Free Lunch, so the condition of No Free Lunch is equivalent to the existence of the extended pricing functional. Stricker [43] proved the theorem without any separability conditions. The general form of the statement is known as the Kreps–Yan separating theorem which plays a crucial role in the semimartingale theory of asset pricing.

Theorem 1 (Kreps–Yan) *Let $G \subseteq L^p$ be a $\sigma(L^p, L^q)$-closed convex cone which includes L^p_-. Assume that $G \cap L^p_+ = \{0\}$. Then there exists a strictly positive linear functional ϕ defined on $L^p(\Omega, \mathcal{F}, \mathbf{P})$ and a $g \in L^q$, satisfying $\phi(f) = \int_\Omega fg d\mathbf{P}$ for each $f \in L^p(\Omega, \mathcal{F}, \mathbf{P})$, and also $\phi(f) \leq 0$ holds true for each $f \in G$.*

The functional ϕ is strictly positive, so the relation $\phi(f) \geq 0$ holds true for each $f \in L^p_+$. We also know that for every f in G the relation $\phi(f) \leq 0$ holds and $0 \in G \cap L^p$, so the hyperspace $\{f \in L^p \mid \phi(f) = 0\}$ is a supporting hyperspace of both set G and the convex set L^p_+, and the hyperspace separates these sets. The most important case is when $p = \infty$. In this case the Kreps–Yan theorem is hardly interpretable because in the case of $p = \infty$ the space $(L^p, \sigma(L^p, L^q))$ is not metrizable, so $\sigma(L^p, L^q)$-closedness is characterizable only with generalized sequences. As will be seen, an important step of Delbaen and Schachermayer's proof is that in the case of a convex cone $\sigma(L^p, L^q)$-closedness can be traced back to some kind of sequential closedness. As will be seen, this problem occurs only in the case of $p = \infty$. The explanation of this is that although the dual of L^1 is L^∞, the dual of L^∞ is not L^1 but *ba*.

In the following let K denote the set of net payoffs available by trading from zero initial wealth. In the case of the Dalang-Morton-Willinger theorem the $K \cap L^0_+ = \{0\}$ – the so called no-arbitrage – condition ensures the existence of the equivalent martingale measure. In this case one can prove that $C_0 \cap L^1$ is closed in L^1. Now, we can use the Kreps-Yan theorem for $p = 1$ while also applying that corollary of the Hahn-Banach theorem which states that the closure of a convex set is the same with respect to topologies compatible with duality. The above principle is applicable to arbitrary $p < \infty$. In these cases we do not need the weak topologies, see *e.g.* Harison–Kreps [25], Duffie [15], and Dallang–Morton–Willinger [9]. Unfortunately in most of the finance applications we have $p = \infty$, so we must use the weak topology.

As mentioned, M. Harrison, D. M. Kreps and S. R. Pliska proved the existence of martingale measures in quite general circumstancies and most of the above results are applicable to diffusion processes (see *e.g.*: [25], [35] and [24]). However, their most important contribution was to recognise the importance of martingale techniques. Among others, they pointed to the fact that the martingale theory fits astonishingly well the problems of option pricing. Among the connections of the two disciplines the most important ones are the results related to the Gisanov transformation and the representation of martingales. The last section of the second chapter demonstrates that by means of the Girsanov transformation the martingale measure can be easily calculated.

The significance of the theory of semimartingales and general theory of semimartingales in finance was firstly pointed out by Harison and Pliska [24]. The general theory of stochastic processes was developed primarily by P. A. Meyer. Thanks to Doob, the theory of martingales, although it did not play an important part, separated from the theory of Markov processes already in the 50s, but until the famous paper by Meyer and C. Doléans-Dade [14] the theory of stochastic integration was not completely independent of it. This latter seminal paper lead to the explosive development of the martingale theory in the 70s and its importance was pivotal with respect to the applicability of stochastic analysis in finance, and also lead to the well-known results of Harrison-Kreps [25] and Harrison-Pliska [24]. The general theory of stochastic processes, independent of the Markov processes, was later developed in the monography by Dellacherie and Meyer [13].

In the third chapters the dissertation displays the extension of the notion of no-arbitrage to multiperiod models and state the fundametal theorem for the case of finite time horizons. This chapter includes the notion of self-financing portfolio and the change of numéraire theorem. This latter states that a self-financing trading strategy remains self-financing after the change of numéraire. In most of the applications the change of numéraire is some riskfree asset, such as a bank account. However, I would like to emphasize that any traded portfolio may be chosen as numéraire.[1] This general form of change of numéraire theorem was proved firstly by Duffie [16] for Itô processes. N. El Karoui, H. Geman and J. C. Rochet [17] extended the theorem for continuous semimartingales, and Shiryaev [42] to discontinuous semimartingale models with bounded variational numéraire. This chapter, applying F. Jamshidian [27], proves the theorem for general semimartingale numéraire.

In chapter four based on [10] I display the entire proof of the Delbaen–Schachermayer theorem. This part of the dissertation is the most detailed, and here can be found the innovative results of the dissertation. Althought I did not succeed in giving a new proof of the theorem, I managed to simplify it in a few points and make the original proof more understandable. The proof of the fundamental theorem for continuous time horizon is rather lengthy and demands enormous efforts. The proof uses deep results of functional analysis and the general theory of stochastic processes developed from the end of the 60s on by the French Probability School, mainly by P. A. Meyer.

In the following, let S be a fixed semimartingale which represents the discounted asset price process. The stochastic integration of a predictable process H with respect to S will be denoted by $H \bullet S$, its value, in a given time, by $(H \bullet S)(t)$ or $(H \bullet S)_t$. A stochastic integral $H \bullet S$ is interpretable as the net result of the trading strategy H if the discounted prices of the assets are given by S. The S-integrable predictable process H is called a-admissible, if for all $t \geq 0$ $(H \bullet S)(t) \geq -a$. The process is admissible if for some positive number a the process is a-admissible. Let $K_0 = \{(H \bullet S)(\infty)\}$ where the limit exists and H is admissible.

As in the discrete time asset pricing, we must assume the set of availaple payoffs to satisfy the free disposal condition, so let C_0 be the cone of the functions dominated by some elements in K_0, that is let $C_0 \doteq K_0 - L^0_+$. Even in the most simple cases of continuous time models, from the Radon-Nykodim derivative of the equivalent martingale measure it is only known that it is an eliment of L^1. This is the main difference between continuous time models and discrete models with finite periods. Since we want to find the Radon–Nikodym derivative by means of separiting and seeking the normal of the hyperspace in L^1, we must restrict the primal space – the space of contingent claims – to the space L^∞, therefore let $C \doteq C_0 \cap L^\infty$. Notice that continous trading enlarges the set of claims available by trade. In this case we must assure the closedness of the set which should be separated. Therefore, we must replace the former algebraic notion of arbitrage with a stronger topological notion which ensures the closedness necessary for the separation. A trivial corollary of the Kreps–Yan theorem is the following. If the semimartingale S

[1] The equivalent martingale measure exists for arbitrary positive semimartingale numeraire, but it depends on the choice of the numéraire. In fact, for arbitrary probability $\mathbf{Q}$, equivalent to $\mathbf{P}$, there exists a trading strategy as numéraire, that the discounted price process is a martingale under $\mathbf{Q}$ (see Conze and Viswanathan [6]). In some models the choice of the numéraire is not unique, so the numéraire may depend on the claim which we want to price. This motivates the so-called change of numéraire method (see: [1]).

satisfies the condition of No Free Lunch, that is to say $\widetilde{C} \cap L_+^\infty = \{0\}$, where $\widetilde{C}$ denotes the $\sigma(L^\infty, L^1)$-closure of the set C, then there exists the equivalent martingale measure. The essence of the Delbaen–Schachermayer theory is, that they managed to verify that it is sufficient to take the much smaller L^∞ closure of the set C, in which case the no-arbitrage condition is much weaker. In the following let $\overline{C}$ be the L^∞ closure of the set C. With these notations we can define Delbaen–Schachermayer's concept of no-arbitrage. The semimartingale S satisfies the condition of No Free Lunch with Vanishing Risk (NFLVR) if $\overline{C} \cap L_+^\infty = \{0\}$. A characterisation of the notion of $NFLVR$ is the following.

Lemma 2 *A semimartingale S satisfies the condition of $NFLVR$ if and only if, for every sequence (g_k) in K_0 the equation*

$$\lim_{k \longrightarrow \infty} \left\| g_k^- \right\|_\infty = 0$$

implies

$$g_k \xrightarrow{\mathbf{P}} 0.$$

In the dissertation I managed to simplify the proof of this crucial lemma. The main statement of the dissertation is the following.

Theorem 3 *Let S be a locally bounded real valued semimartingale with respect to $\mathbf{P}$. There is a probability $\mathbf{Q}$ equivalent to $\mathbf{P}$ under which S is a local martingale if and only if S satisfies $NFLVR$.*[2]

The non-trivial direction of the theorem is the proof of the existence of the equivalent martingale measure. The main difficulty of the proof is summarized in the following theorem.

Theorem 4 *Let S be a bounded real valued semimartingale which satisfies the condition of NFLVR. Then cone C is $\sigma(L^\infty, L^1)$-closed in L^∞.*

In the following let B_∞ be the closed unique ball of L^∞. The key step of the previous theorem is the following corollary of the Krein–Šmulian theorem.

Theorem 5 *A convex cone C in $L^\infty(\Omega, \mathcal{F}, \mathbf{P})$ is $\sigma(L^\infty, L^1)$-closed if and only if set $C \cap B^\infty$ is closed with respect to the stochastic convergence.*

Delbaen and Schachermayer (see: [10] and [12]) traced the theorem back to the Mackey–Arens theorem and the Grothendieck lemma. One interesting result of the dissertation is that it gives a much more elementary proof of this theorem which does not use the difficult notion of the Mackey topology.[3]

Delbaen and Schachermayer [11], using a quite different approach, extended the fundamental theorem to the case of unbounded semimartingale price process. In this case the condition of $NFLVR$ is equivalent to the existence of an equivalent σ-martingale measure.

The concept of martingale measure plays an important role not only in pricing derivatives but in the theory of porfolio optimization in incomplete markets. It is well known that for non-Markovian diffusion processes the optimal control methods of portfolio optimization do not work, so, from the middle of the 80s on, Pliska [38], Karatzas *et all* [33] and

[2] The statement can be easily extended to locally bounded semimartingales.

[3] Published: A pénzügyi eszközök árazásának alaptétele lokálisan korlátos szemimartingál árfolyamok esetén, Szigma, 2009/3–4. After the publication of the result we were informed by Miklós Rásonyi that Kabanov has a similar, presumably unpublished proof.

Cox and Huang [7] developed the so-called martingale – or duality – methods of portfolio optimization. The point here is that dynamic optimization problems can be transformed into static optimization problems which, by using duality techniques, are easier to solve. After having resolved a static optimization problem the optimal trading strategy can be obtained by the application of the martingale representation theorem. The investigation of the mentioned duality techniques was the purpose of Bellini and Frittelli's article. They assert a duality theorem which constitutes the base of the new version of the FTAP.([20]). In Chapter Five the dissertation offers a short introduction to the duality approach of portfolio optimization, then in Chapter Six, departing from the results of Bellini and Frittelli following the articles [20], [22] and [34], it presents the complete proof of Frittelli's fundamental theorem of asset pricing. In Chapter Seven, after overviewing the necessary fundamentals on Orlicz spaces, following the articles of [20], [21] and [34], it shows that the most important concepts of no arbitrage can be characterized by means of the preferencies of the investors. The main message of this chapter is that the mentioned results of Frittelli and Klein throw new light upon the Delbaen–Schachermayer theory economically. To see this, we turn to the relationship between the concept of no-arbitrage and the preferencies of the investors.

It is said that the condition of no-arbitrage implicitly presupposes that the preferences of the investors are monotonous. We have also seen that the no free lunch condition is equivalent to the viability which presupposes convex preferencies. This raises the question of whether there is a unified conceptual framework by means of which the difference between the no-arbitrage concepts can be traced back to the discrepancies between the preferencies.

It would seem to be a good solution to use the notion of stochastic dominance. The relationship between stochastic dominance and arbitrage was investigated by R. Jarrow [28]. He proved that the existence of arbitrage in a complete market is equivalent to the existence of two assets with a special property one of which is stochastically dominated by the other.

Frittelli's notion of "no market free lunch" is based on the notion of stochastic dominance. This concept, instead of topological concepts involved in various arbitrage concepts, uses the analitical property of the investor's utility functions. Let us fix some utility functions. Suppose that, using zero initial wealth, you can obtain a random payoff f which takes the form of $w + q$, where w is non-negative but strictly positive on some positive measured set and q is a random payoff the utility of which is greater than or equal to the zero payoff with respect to any monotonous utility functions. In this case there is a kind of arbitrage possibility called Market Free Lunch.

As mentioned, the condition of no-arbitrage, in general, does not imply the existence of the equivalent martingale measure, so we need a stronger condition, that is, we have to enlarge the set of excludable arbitrage possibilities. Taking into account the above approach, this enlargement can be done by requiring the above property of q with only a narrower set of utility functions. This is the point where the Delbaen–Schachermayer and the Frittelli approaches separate. From the characterisation of these "no-arbitrage" concepts it follows, that the former, besides monotonity, only requires continuity while the latter requires the concavity too. Klein's characterisation of the No Free Lunch implies that Frittelli's fundamental theorem is similar in depth to the Kreps–Yan theorem, and the equivalence of the two statements is easy to prove by means of Orlicz space methods. Because both the Delbaen–Schachermayer theorem and Frittelli's fundamental theorem state an equivalence between their respective no-arbitrage concepts and the existence of a local martingale measure, the two arbitrage concepts are equivalent. Therefore, let's assume that the investors have concave utility functions and that the market is consistent; then this market already does not provide new arbitrage oportunities for investors with continuous but not necessarily concave utility functions. So, an interesting message of the Delbaen–Schachermayer theory is that while investigating the consistency of the market the concavity of the investors' utility function does not mean any restriction. Notice the analogy between one of the known results of duality theory of microeconomics. The duality principle of the production theory asserts that the cost function includes all the economically important properties of the technology. In fact, if we fix an arbitrary cost function, there is always a convex input requirement set to which it can be traced back. That is, the convexity of the technology is not too restrictive assumption. However, the proof of this latter well-known statement of economics uses simple convex analysis, while the proof of our statement – as it depends on the special structure of cone C – probably cannot be proved without the general theory of stochastic processes.

Finally, Chapter Seven deals with the law of one price, its characterisation by means of the notion of state price deflator, and the relationship between the law of one price and the condition of no-arbitrage.

3 Directions for Further Research

As mentioned, the subject of the dissertation is closely related to the second fundamental theorem of asset pricing, that is to the problem of completeness which has an extensive literature (see *e.g.* [10], [4] and [5]). The investigation of this field is a possible research task.

From the point of view of applications, the Lévy processes constitutes an important class of semimartingales. As mentioned in the introduction, the Wiener process theory of asset pricing is already rather well developed. There are many textbooks dealing with this theory. However, up to now, there has hardly been any books or monographies about the Lévy process approach of asset pricing. It is an empirical fact that Lévy processes provide a better description of asset prices than the Wiener process models, so the investigation of these models are of high importance.

The dissertation assumes that there are no costs of trading, that is to say, the market is frictionless. One of the most important results of the last decade is that in some types of models this assumption can be abandoned, and the fundamental theorem can be extended to some class of models with frictions (for discrete models see: [31] and [41], for continuous models see: [23]). A possible future research task is the investigation of these models and the extension of the results to semimartingale models.

Finally, an interesting and promising research field is the gauge theory approach of financial markets. This latter theory is based on the fact that there is a differential geometric characterisation of the no-arbitrage condition. Although this approach is not very new (see [26]) it is not well-known. The cause of this may be the novelty and the unusual nature of the used mathematical apparatus. The investigation of this theory would be interesting not only from the point of view of asset pricing but from the point of view of its application to economics. Although some economists hope gauge theory to revolutionize the theory of economics, as we know, gauge theory is only used in the theory of arbitrage (see: [26], [44], [19] and [18]) and in price index theory (see: [36]).

4 Publications

1. Szemimartingálok elmélete és a pénzügyi eszközök árazásának alaptétele, I. Országos Gazdaság és Pénzügyi Matematikai PhD Konferencia kiadványa, 2008, Budapest

2. On the General Mathematical Theory of Asset Pricing: Duality in Finance and the Fundamental Theorem of Asset Pricing, (coauthor: Medvegyev Péter), 16th International Conference on Mathematical Methods in Economy and Industry, 2009, (Joint Czech-German-Slovak Conference, Ceské Budejovice)

3. A pénzügyi eszközök árazásának alaptétele lokálisan korlátos szemimartingál árfolyamok esetén, Szigma, 2009/3–4., coauthor: Medvegyev Péter

4. Topológikus vektorterek az arbitrázselméletben, Szemináriumi előadás, Pannon Egyetem Matematika Tanszékének és a VEAB Matematikai és Fizikai Szakbizottsága Matematikai Analízis és Alkalmazási Munkabizottságának szervezésében, 2010

References

[1] Bajeux-Besnainou, I. – Portrait, R.: The Numeraire Porfolio: A New Perspective on Financial Theory, The European Journal of Finance, 3 (1997), 291–309.

[2] Bellini, F. – Frittelli, M.: On the Existence of Minimax Martingale Measures, Math. Finance, 12/1 (2002), 1–21.

[3] Bewley, T.: Existence of Equilibria in Economies with Infinitely Many Commodities, *Journal of Economic Theory*, 4 (1972), 514–540.

[4] Cherny, A.S.: Vector Stochastic Integral in the First Fundamental Theorem of Asset Pricing, Proceedings of the Workshop on Mathematical Finance, INRIA, (1998), 149–163.

[5] Cherny, A.S. - Shiryaev, A. N.: Vector Stochastic Integral and the First Fundamental Theorem of Asset Pricing, Proceedings of Steklov Mathematical Institute Seminar, 237 (2002), 12–56.

[6] Conze, A. – Vishwanathan, R.: Probability Measues and Numeraire, CEREMADE, Université de Paris, 1991

[7] Cox, J. C. - Huang, C. F.: Optimal Consumption and Portfolio Policies when Asset Prices Follow Diffusion Processes, J. Econ. Th. 49 (1989), 33–83.

[8] Cox, J. C. –Ross, S. – Rubinstein, M.: Option Pricing: A Simplified Approach, Journal of Financial Economics, 3 (1979), 145-166.

[9] Dalang, R. C. – Morton, A. – Willinger, W.: Equivalent martingale measures and no-arbitrage in stochastic securities market model. Stochastics Stochastics Rep., 29 (1990), 185–201.

[10] Delbaen, F. – Schachermayer, W.: A General Version of the Fundamental Theorem of Asset Pricing, Math. Ann., 300 (1994), 463–520.

[11] Delbaen, F.-Schachermayer, W.: The Fundamental Theorem of Asset Pricing, for Unbounded Stochastic Processes, Math. Ann., 312 (1998), 215–260.

[12] Delbaen, F.-Schachermayer, W.: The Mathematics of Arbitrage, Springer-Verlag, Berlin, 2006

[13] Dellacherie, C. - Meyer, P. A.: Probabilités et potentiel, Hermann, Paris, vol I, Vol II, 1976, 1980

[14] Doléans-Dade C. - Meyer, P.A.: Intégrales stochastiques par rapport aux martingales locales, Séminaire de Probabiliés IV, Lecture Notes in Mathematics, 124 (1970), 77–107.

[15] Duffie, D.–Huang, C–f.: Multiperiod Security Markets With Differential Information, Journal of Mathematical Economics, 15 (1986), 283–303.

[16] Duffie, D.: Dynamic Asset Pricing Theory, Princeton University Press, 1992

[17] El Karoui, N. – Geman, H. – Rochet, J. C.: Changes of numéraire, changes of probability measure and option pricing, Journal of Applied Probability 32 (1995), 443–458.

[18] Farinelli, S.: Geometric Arbitrage and Market Dynamics, preprint, elérhető: www.ssrn.com, 2009

[19] Farinelli, S.–Vazquez, S.: Gauge invariance, Geometry and Arbitrage, preprint, elérhető: www.ssrn.com, 2009

[20] Frittelli, M.: Some Remarks on Arbitrage and Preferences in Securities Market Models, Math. Finance, 14/3 (2004), 351–357.

[21] Frittelli, M.: No arbitrage and Preferences, Instituto Lombardo – Accademia di Scienze e Lettere, (2007), 179–199.

[22] Frittelli, M. – Lakner, P.: Almost Sure Characterisation of Martingals, Stochastics and Stochastics Report, 49 (1994)

[23] Guasoni, P.–Rásonyi, M.–Schachermayer, W.: The Fundamental Theorem of Asset Pricing for Continuous Processes under Small Transaction Costs, online: Anals of Finance, 2008

[24] Harrison, J. M. – Pliska, S. R.: Martingales and Stochastic integrals in the Theory of Continous Trading. Stochastic Process. Appl., 11 (1981), 215–260.

[25] Harrison, J. M. – Kreps, D. M.: Martingales and Arbitrage in Multiperiod Securities Market. J. Econom. Theory, 20 (1979), 381–408.

[26] Ilinski, K.: Physics of Finance, Wiley, New York, 2001.

[27] Jamshidian, F: Numeraire Invariance and Application to Option Pricing and Hedging, MPRA Paper, No. 7167, February 2008

[28] Jarrow, R.: The Relationship between Arbitrage and First Order Stochastic Dominance, The Journal of Finance, vol. XLI, no. 4, (1986), 915-921.

[29] Jones, L. E.: Special Problems Arising in the Study of Economies with Infinitely Commodities, In:Models of Economic Dynamics, ed.: Hugo, F., Springer-Verlag, Sonnenscheim, Berlin, 1986

[30] Kabanov, Yu. M.: On the FTAP of Kreps–Delbaen–Schachermayer, Statistics and Control of Random Processes, The Liptser Festschrift, Proceedings of Steklov Mathematical Institute Seminar, (1997), 191–203.

[31] Kabanov, Yu. M.–Rásonyi, M.–Stricker, C.: No-arbitrage Criteria for Financial Markets with Efficient Frictions, Finance Stoch., Vol. 6 (2002), pp. 371–382.

[32] Kabanov, Yu. M. – Stricker, C.: A Teachers' Note on No-arbitrage Criteria, Séminaire de probabilités (Strasbourg), 35 (1997), 149–152.

[33] Karatzas I. – Lehoczki, J. – Shreve, S. – Xu, G.:Martingale and Duality Methods For Utility Maximization in an Incomplete Market, SIAM J. Control Optim., 29 (1991), 702–730.

[34] Klein, I: A comment on market free lunch and free lunch, Math. Finance 16 (2006), 583–588

[35] Kreps, D. M.: Arbitrage and Equilibrium in Economies with Infinitely Many Commodities, J. Math. Econom., 8 (1981), 15–35.

[36] Malaney, P. N.: The Index Number Problem: A Differential Geometric Approach, PhD Thesis, Harvard University Economics Department, 1996

[37] Péter Medvegyev: *Technical results for no-arbitrage theorems in continuous time*, Kézirat, 2007

[38] Pliska, S. R.: A stochastic calculus model of continuos trading: optimal portfolios, Math. Oper. Res., 11, (1986), 371–382.

[39] Prescott, E. C. – Lucas, R. E.: Price systems in Infinite Dimensional Space, International Economic Review, 13 (1972), 416–422.

[40] Schachermayer, W.: A Hilbert Space Proof of the Fundamental Theorem of Asset Pricing in Finite Discrete Time, *Insurance Math. Econ.* 11 (1992), 249–257.

[41] Schachermayer, W.: The Fundamental Theorem of Asset Pricing Under Proportional Transaction Costs in Finite Discrete Time, Math. Finance, Vol. 14 (2004), pp. 19–48.

[42] A. N. Shiryaev: Essentials of Stochastic Finance: Facts, Models, Theory, World Scientific, Singapore, 2001

[43] Stricker, C.: Arbitrage et Lois de Martingale, Annales de l'Institut Henry Poincaré – Probabilités et Statistiques, vol. 26 (1990), 451–460.

[44] Young, K.: Foreign Exchange Market as a Latice Gauge Theory, Am. J. Phys. 67, 1999