ABSTRACT OF THE Ph.D. THESIS

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Ph.D.

APPLICATION OF MATHEMATICAL AND STATISTICAL METHODS TO AGRICULTURAL DECISION MAKING

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CONTENTS

1 PRELIMINARIES, AIM OF STUDY

By the end of the 80s fundamental changes occurred in several spheres of the Hungarian economy; market factors started to play a significant role in the economy. In this period the agriculture, especially the food-industry went through a great transformation.

With the appearance of market factors, sufficient information and uncertainty become important factors in the economy. Decision makers find themselves in a entirely new situation, due to the increased level of uncertainty.

As external and environmental uncertainty (e.g. uncertainty due to changing weather conditions and due to political interventions) is present to a higher extent in the agriculture than in other fields of economy, agricultural decisions take place under exceptionally high degree of uncertainty.

Agriculture is a special field of economy since it has a great importance in the production of fundamental consumption goods. As a consequence, the proper treatment of uncertainty in these decisions is a strategic matter.

In the presence of uncertainty, especially the novel decisions are of a high-risk because wrong decisions may have very bad consequences on the company. Therefore, one should consider all possible methods and approaches for the support of decision making

Under uncertainty one cannot rely on traditional mathematical and statistical tools. Therefore, there is a great need for a research on the development and application of new mathematical and statistical methods that can be applied to support decision making under uncertainty and insufficient information. In addition to this, the analysis of decisions in the food-industry is also a central task.

To solve the above mentioned problems the focus of the thesis was on the application (and on proving the usability) of the *Bayesian decision* model, the *Bayesian statistics, and* the *maximum entropy* in the case of uncertain situations with incomplete information.

The main goal of the thesis is to show that it is possible to make proper decisions even when the decision-maker has only incomplete information available and the decision needs to be taken under uncertainty.

Further goals of the dissertation are to accept or to reject the following hypotheses:
Under uncertainty and incomplete information
(1) for *investment decisions* the *Bayesian decision model*,
(2) for the *design of a new product* the *Bayesian statistics*, and
(3) for *price decisions* the *maximum entropy*
are the appropriate methods for supporting decision-making.

To assessment of decisions in the food industry the thesis tried to find answers to the following questions:

1. What are the main characteristics of food-industry decisions?

2. To what extent are these decisions successful and what are the reasons for unsuccessful decisions?

3. To what extent are decisions in the food-industry subject to uncertainty and improper information?

4. Are mathematical and statistical methods applied in the decision making in the food-industry?

5. Does the application of the Bayesian decision model, the Bayesian statistics, and the maximum entropy lead to better decision under uncertainty and improper information?

2 SUBJECT AND METHOD

The research that lead to this thesis can be classified into the following stages:

1st stage: Continuous treatment of literature.

2nd stage: Questionnaire and dept interview and processing of collected data.

3rd stage: Adaptation and application of the Bayesian decision model, Bayesian statistics, and the maximum entropy and preparation and presentation of their algorithms.

4th stage: Data collection for proving the adequacy and applicability of the Bayesian decision model, the Bayesian statistics, and the maximum entropy.

The distinct stages of the research required different approaches.

1. *During the continuous treatment of literature* the situation of agriculture, the national and international literature on mathematical and statistical methods for decision-support and results were surveyed.

2. *The questionnaires* aimed to explore (1) how decision making takes place in reality, (2) whether there are indeed managerial decisions that

need to be taken under uncertainty and incomplete information, (3) how much do uncertainty and incomplete information affect decision-making, and (4) whether decisions could be improved with the support of appropriate mathematical and statistical methods.

I made two surveys, one in 1993 and one in 2000. In the first case a causal, while in the second case a stratified data collection method was applied. Although the method and the main focus of the two questionnaires were not exactly the same, they made it possible to draw some conclusions and make some comparisons in connection with decision-making and quantitative methods.

The depth-interview, aimed at managers of leading food-product companies and at those employees of market research companies who apply quantitative methods, helped to get a better understanding of the information obtained from the surveys. With its help I could identify the areas where the applied methods and approaches are unsatisfactory and where there is an urgent need for new mathematical and statistical methods.

3. *Before working out the adaptation of the algorithms* an essential point was to clarify how Bayesian statistics handles the concepts and methods known from “classical” statistics and the substance of maximum entropy.

4. To prove the applicability of the data collected from the surveys, and the to *verify the usability of the algorithms* a primer and a secunder data

collection was required. In this stage finding appropriate software and the adaptation of the methods to the given problem meant the biggest difficulty.

3 RESULTS

Bayesian statistics and maximum entropy are quite unknown methods in Hungary, one may rarely meet these methods even in the scientific literature. Their application for market decisions is very rare.

Results of questionnaires and depth-interviews

Managers think that higher profitability can mainly be achieved through better consumer satisfaction. In obtaining this, insufficient information and uncertainty provides the greatest difficulty (99%).

Based on the results of the linear regression model with one dummy variable the following can be concluded. The success of a decision depends only on whether quantitative methods are applied in the preparation. However, this explains the success only to a quite limited extent. We can still conclude that those decisions that are supported with quantitative methods are more successful than those which are not.

In our application in which we used a probit-model, the novelty of the decision and the existence of a decision-support group are the only factors that significantly influence the application of quantitative methods for decision-making. The higher the novelty of the decision, the wider the application of mathematical and statistical methods in the decision-making. In addition, the existence of a decision-support group has a positive influence on the usage of quantitative tools.

According to the questionnaires and depth-interviews the need for a new method became clear.

Several managers of companies and market-researchers were asked in depth-interviews for possible reasons for uncertainty and for need of new quantitative methods. The managers (Table 2.) named three important fields where the need for new methods is a essential; *investment decisions*, *introduction and design of new products*, and *price decisions*. All these problem arise either from lack of appropriate data or multicollinearity. Table 1. presents the answers from market-researchers.

Table 1.

Summary of demand for new methods at market-researchers

Type of researcher	Reasons for uncertainty	Applied quantitative methods	**Demand for new method**
1.	Incomplete data.	Mainly data-mining, non-linear fitting. cluster-analysis, discriminant analysis.	**Multicollineariy**
2.	Lack of appropriate method in the case of incomplete data.	Log-linear analysis, structural equations, cluster-analysis, discriminant analysis, conjoint analysis.	**Insufficient data and multicollineariy.**

Table 2.

Summary of demand for new methods at companies

Type of company	Reasons for uncertainty	Applied quantitative methods	**Demand for new method**
1.a.	Fluctuation of stock prices and world market prices, uncertainty in agriculture, dependency on the foreign main company.	Our company has no insight into quantitative methods, but we apply regular market-research. It is often the main company that provides the results.	**Price decisions.**
1. b.	Besides the above mentioned, high competition on the domestic market.	Descriptive statistics, game theory, trends, correlation and regression analysis, and elasticities. No insights into methods applied for market research.	**Price decisions, introduction of new products, when only total costs of activities is known.**
2.	Dependency on the agriculture and networks, lack of regulation, over-developed capacity, and export market becoming smaller.	Descriptive statistics, cannot afford a market research company.	**Price decisions, investigation of customer needs, and introduction of new products.**
3.	All fields	None	**None**

In the case of Table 1. the type of researchers are the following:

1. Those research companies that provide their services to a multinational company.

2. The "classical" type of market research companies, who have a broad range of customers.

In Table 2. the types of companies are the following:

1. a. Those *multinational* companies that are still growing rapidly,

1. b. Those *multinational* companies whose growth has somewhat slowed down,

2. Successful medium-sized companies,

3. Small businesses.

The questionnaires and depth-interviews revealed that neither the managers of companies, nor the market-researchers are acquainted with the Bayesian decision model, Bayesian statistics, or maximum entropy.

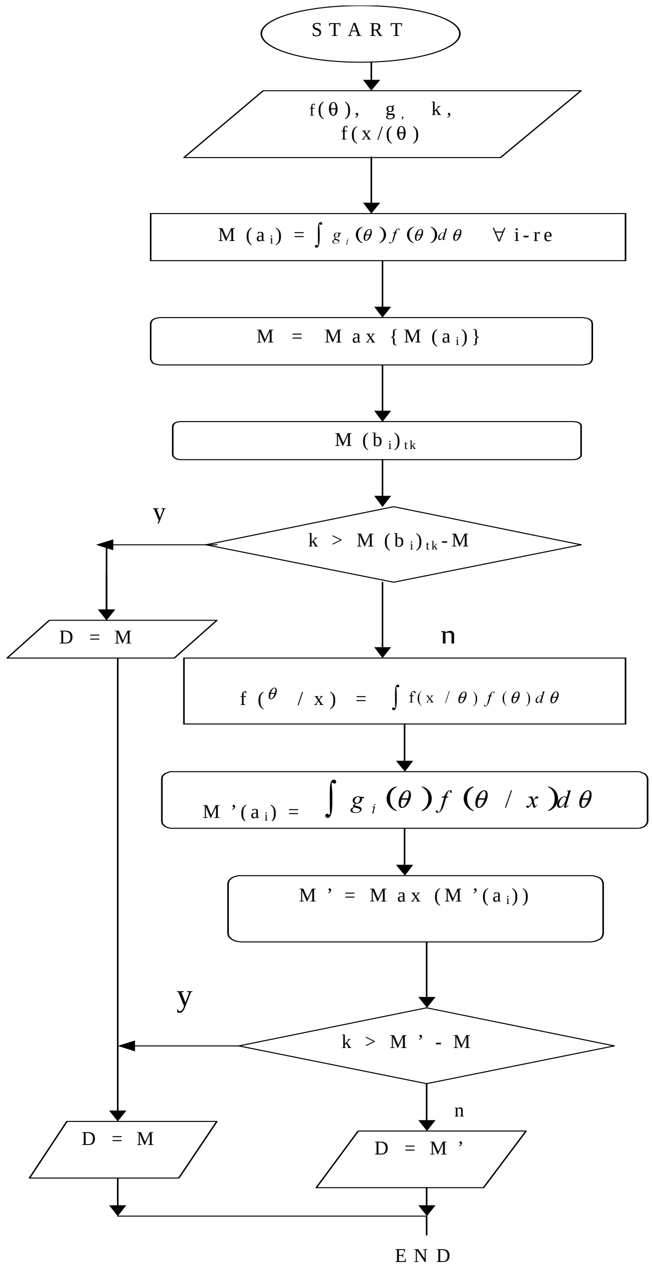
The Bayesian approach and maximum entropy approach has a lot in common. These common features make them able to handle situations when insufficient information is available or multicollinearity is present. These approaches are capable of using all available information. Bayesian statistics does so by using subjective probability-based a priori knowledge and maximum entropy does so by applying the maximum entropy principle.

Results of the application of the Bayesian decision model

For the support of investment decisions under uncertainty the following algorithm of the Bayesian decision model (Figure 1.) is suggested:

Figure 1.

The algorithm of Bayesian decision for continuous variables, extended by decision-making on collecting information



Notations in Figure 1.:

$f(\theta)$: a priori density function,

$f(x/\theta)$: a Likelihood function

g: profit function,

K: cost of the market researchers and of sampling

$M(a_i)$: expected profit based on only a priori information,

$M(b_i)_{tk}$: expected profit in the case of perfect information,

$f(\theta/x)$: a posterior density function

During the application of the algorithm for Bayesian decision model it became clear that the suggested algorithm can be successfully applied, in the case of both discrete and continuous variables, to support decisions about the employment of a market research company.

In addition, the application confirmed that the Bayesian decision model can be applied for investment decisions concerning the production of new products. It makes it possible to involve market research into these decisions. It enables the managers to determine the maximum amount that is worth paying for market research. Knowing the reliability of the market research company the manager becomes capable of deciding whether or not to make use of the offered services and if she/he decides to do so, what would be the optimal payment in the case of different predictions. With he use of the Bayesian decision model the company manager can determine all these before making use of any service of the market research company.

Results of the application of Bayesian statistics

The following version of the hierarchical random coefficient model is a suitable tool to assess consumer heterogeneity and consumer preferences:

$$Y_i = X_i \beta_i + \varepsilon_i$$
$$\beta_i = \theta z_i + \delta_i$$

n: number of observations,

Y_i : a J_i dimensional vector of responses for subject i to the profiles,

X_i : a T_i x p dimensional design matrix of the dummy variables,

β_i : a p-dimensional vector of regression coefficients (part-worth for subject i)

In the second equation:

θ: a pxq matrix of regression coefficients,

z_i: a q dimensional vector of covariances.

Furthermore, assume the following is assumed:

$\{\varepsilon_i\},\{\delta_i\}$ are mutually independent and they have multivariate normal distribution:

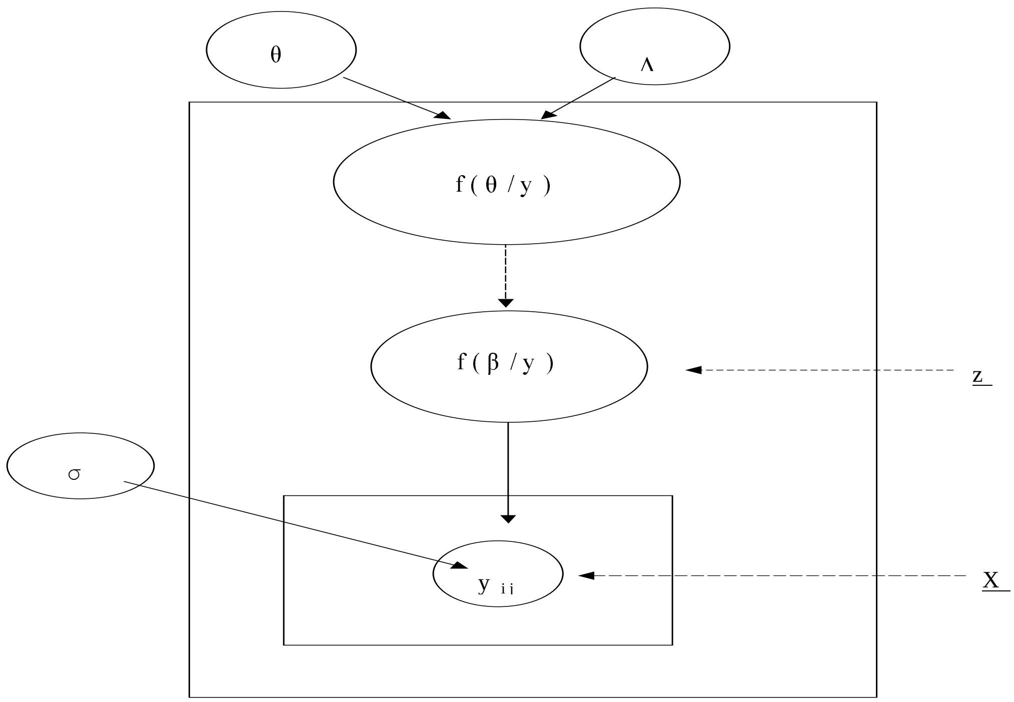
$$\varepsilon_i \sim N_{J_i}\left(0, \sigma_i^2 I_{J_i}\right)$$
$$\delta_i \sim N_p(0, \Lambda)$$

where I_{T_i} is a $J_i x J_i$ identity matrix and Λ is a pxp positive definite matrix.

The suggested algorithm is can be seen on the following figure:

Figure 2.

The Random Coefficient Hierarchical Bayes model



During the application of the developed algorithm the we randomly drop (two-by-two) a hypothetical product from the data. We find that using the Bayesian algorithm we can achieve results which are close to that of Ordinary Least Squares (OLS) in precision, in the path-values (part-worth) (r=0,93) even in the case of missing data, when several customers did not rank some of the profiles. When the number of profiles was decreased to half of the original the estimated preferences still had a medium-sized positive correlation (r=0,616) with those obtained with OLS. This shows that it is possible for marketing researchers to get quite precise results when investigating consumer preferences if the respondents do not want or cannot value all the products.

Results of the application of maximum entropy

The suggested approach for measuring demand price elasticity in the presence of multicollinearity is the following non-linear regression model:

$$\ln q_{it} = \alpha_i + \sum_{1}^{K} \beta_{ik} \ln p_{kt} + \gamma_i \ln\left(\frac{M_t}{P_t}\right)$$

i: number of equations: (i=1,...N)

t: number of observations (t=1,...T)

k: number of unknown price parameters (k=1...K)

q_{it}: consumption per person

p_{kt}: unit price of the products

$M_t = \sum_{1}^{K} p_{kt} q_{kt}$

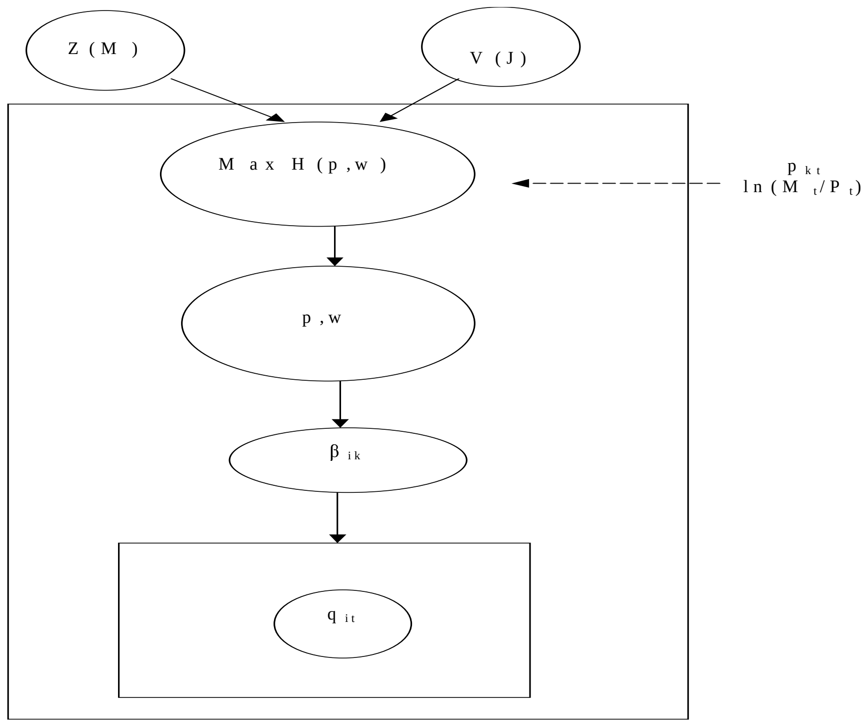
P_t: Ston's index, where

$$\ln P_t = \sum_{k=1}^{K} \frac{p_{kt} q_{kt}}{\sum_{k=1}^{K} p_{kt} q_{kt}} \ln p_{kt}$$

The suggested algorithm can be seen on Figure 3.

Figure 3.

Algorithm to measure demand price elasticity in case of multicollinearity



The results of the application of the general maximum entropy, a special case of maximum entropy principle, seem reasonable (have high face-validity). This suggests that cross price elasticities can even be measured for very close substitute products, such as oil, fat, and butter. Multicollinearity can be overcome with the use of the designed algorithm.

Summing up, the above introduced algorithms and approaches provide quantitative methods for decision support in the case of uncertainty, insufficient data, and multicollinearity.

4 NEW SCIENTIFIC RESULTS

1. The questionnaires and the depth-interviews showed that lack of information and uncertainty mean the greatest difficulty for the decision-makers.

2. The application of quantitative methods influences the success of the decision significantly. The existence of a decision-support group has a positive influence on the usage of quantitative tools. In addition, the higher the novelty of the decision, the wider the application of mathematical and statistical methods in the decision-making.

3. At companies the demand for new methods occurs mainly in the case of investment decisions, price decisions, introduction of new products, and break-down of the costs of full activities. This need arises from uncertainty and improper information in all cases.

4. Traditional mathematical and statistical tools cannot provide appropriate support for decision making in the presence of the above mentioned problems.

5. Several similar characteristics can be found in the Bayesian approach and the maximum entropy approach that enable them to handle uncertainty.

6. Bayesian decision model, Bayesian statistics, and maximum entropy, with appropriate adaptation and with the development of new algorithms, provide adequate tools for managers and market-researchers to solve the above mentioned problems.

7. The Bayesian decision model offers support for investment decisions related to the introduction of a new product.

8. The adaptation of the Hierarchical Bayes model makes the application of conjoint analysis possible, even when traditional approaches do not work, namely, when the respondents do not value all the offered hypothetical products.

9. General maximum entropy renders it possible to analyse the price elasticity of the demand even for products that could only be examined imprecisely or could not be analysed at all with traditional methods, due to multicollinearity problems.

10. Bayesian estimation combined with general maximum entropy estimation is expected to lead to better decisions.

5 FURTHER AREAS FOR RESEARCH

The study of consumer heterogeneity at more detail when additional questions about the characteristics of the customers are available, with the use of the Random Coefficients model. The investigation of how much this additional information improves the conjoint results.

Adaptation of Mixture models for the analysis of consumer preferences.

Application of cross-entropy, which arises from the joining of Bayesian statistics and maximum entropy, to a concrete problem.

Bayesian approach to time series analysis.

Broader application of the methods, e.g., in other research on consumers.

The application of a priori information in game theoretic models.

Application of the suggested methods in reality, usage of these methods in companies' decision-making.

Provision of appropriate software for application of methods introduced in the thesis.

6 LIST OF PUBLICATIONS OF THE AUTHOR

1. Modell-módszer alkalmazása a fizikában. Egyetemi doktori disszertáció, JATE BTK, Szeged, 1984.

2. Bayesi szemléletváltás. (Szerk.: Temesi, G.). „Európai integráció és felsőoktatás” Jubileumi nemzetközi tudományos konferencia, KVIF, 1999. 125. p.

3. Bayesi döntések. (Szerk.: Polgár, J. P.-Tóth, I.). VI. Ifjúsági Tudományos Fórum, Veszprémi Egyetem Georgikon Mezőgazdasági Tudományi Kar, Keszthely, 2000. 1-7. p. (CD)

4. A racionalitás problémája és a játékelmélet legújabb eredményei. (Szerk.: BGF KKFK Tudományos Tanácsa – Majoros, P.). Szakmai Füzetek 9., BGF KKFK, Budapest, 2001. 98-105. p.

5. Bayes tétel alkalmazása a conjoint analízisben. (Szerk.: BGF KKFK Tudományos Tanácsa – Majoros, P.). Szakmai Füzetek 10., BGF KKFK, Budapest, 2001. 27-45. p.

6. Application of Bayesian Methods in Conjoint Analysis. (Ed. and lect.: Szűcs, I.). 1st International Conference for Young Researchers, Szent István Egyetem, Gödöllő, 2001. 149-158. p.

7. A Bayes statisztika és alkalmazása. (Szerk.: Majoros, P.). Tudományos Évkönyv, BGF, Perfekt, 2001. 218-226. p.

8. Dönteni bizonytalanság körülményei között. (Szerk.: Majoros, P.). Tudományos Évkönyv, BGF, Perfekt, 2002. 341-349. p.

Presentations at scientific conferences

1. A racionalitás problémája és a játékelmélet legújabb eredményei. „Vállalkozáselmélet módszertani problémái a gazdasági felsőoktatásban" című konferencia. PSZF, Budapest, 1997.

2. Bayesi döntéselmélet. „Főiskolai Matematika-, Fizika- és Informatikaoktatók XXIII. Országos Konferenciája". Miskolci Egyetem Dunaújvárosi Főiskolai Kar, Dunaújváros, 1999.

3. A maximum entrópia elv alkalmazása hiányos és nem teljes információk esetén. „Az elemzési módszerek és a gazdasági környezet harmonizációja." Tudományos konferencia, BGF, 2000.

4. Bayes tétel alkalmazása a conjoint analízisben. „Operációkutatás és alkalmazott statisztika" című tudományos ülés. BGF KKFK, 2001.

5. A fogyasztói heterogenitás vizsgálata a hierarchikus Bayes módszer alkalmazásával. „Főiskolák Matematika, Fizika, Számítástechnika Oktatóinak XXV. Országos Konferenciája”. BGF PSZFK, Zalaegerszeg, 2001.

6. Application of Bayesian Methods in Conjoint Analysis. „1st International Conference for Young Researchers”, Szent István Egyetem, Gödöllő, 2001.

7. A Bayes statisztika és alkalmazása. „Globalitás és vállalkozás” tudományos konferenciasorozat a Magyar Tudomány Napja 2001 tiszteletére, BGF, 2001.

8. Application of Maximum Entropy methods in case of scarce and incomplete information. „Főiskolák Fizika, Matematika és számítástechnika Oktatóinak XXVI: Országos Konferenciája”. Berzsenyi Dániel Főiskola, Szombathely, 2002.

9. A vállalatok döntés-előkészítési módszerének elemzése. Módszertani szimpózium, BGF KVIF, Budapest, 2002.

10. Döntés bizonytalanság körülményei között. „Híd kelet és nyugat között” Tudományos konferenciasorozat a Magyar Tudomány Napja 2002 tiszteletére, BGF, 2002.

Other published materials

1. Gazdasági matematika I. Feladatgyűjtemény. (Szerk.: Madaras, Lné.) KGF, Szolnok, 1996. IV., V. fejezet.

2. Távoktatási útmutató a Gazdasági matematika I. tantárgyhoz. Phare, (Lekt.: Tóth, I.) Student Kiadó, Szolnok, 1997. 4., 5., 7. fejezet.

3. Távoktatási útmutató a Gazdasági matematika II. tantárgyhoz. Phare, (Lekt.: Tóth, I.) Student Kiadó, Szolnok, 1998. 4., 5. fejezet.

4. Feladatgyűjtemény a Statisztika I., II. jegyzethez. (Szerk és lekt.: Horváth, Gné.).BGF KKFK, Budapest, 2002. I.5. I.6. I.7. II.1. II.5. II.6. fejezet.