

**COMPUTER AIDED NUMERICAL  
ANALYSIS OF THE CONTINUOUS  
GRINDING PROCESSES**

Theses of PhD Dissertation

Written by

**PIROSKA BUZÁNÉ KIS**

Information Science PhD School  
University of Veszprém

Supervisors: Zoltán László  
Csaba Mihálykó

Department of Mathematics and Computing  
University of Veszprém  
Hungary

2004

## Preliminaries and goals

Grinding plays an important role in number of fields of the industry. The grinding is great energy consuming process. According to the large scale applications, the grinding devices are very different from each other regarding ones operation modes and sizes. The object of the dissertation is the investigation of the continuous grinding process carried out in (cylinder) ball mills. There are two main types of the continuous grindings: the open circuit and the closed circuit grindings. During the open circuit grinding the material to be ground is continuously fed into the mill at the inlet of the mill. The material flows along the mill and is being grinded, then the ground material leaves continuously through the outlet of the mill. The grinding mill is often coupled with a classifier in the practice. In this case the ground material is classified. The fine part of the ground material makes the product of the grinding and one is taken away from the grinding process, whilst the coarse part is returned into the mill. This type of grinding is named as closed circuit continuous grinding. During the closed circuit grinding it might occur overload, overcharge of the mill as a heavy breakdown which must be avoided.

The mathematical description of the continuous grinding processes is rarely insufficient because of the difficulties of the problem. Computer aided analysis of the process offers the solution. Comparing to the open circuit grinding, the mathematical description of the closed circuit continuous grinding is even more difficult because of the considering the classifier. Regarding to the closed circuit continuous grinding, neither analytical studies, nor numerical investigations have been executed. The mathematical models for these purposes have been found in very early states of their developments.

The goal of the dissertation is to present the mathematical model of the closed circuit continuous grinding that meets the following requirements:

- The model involves the description of the open circuit continuous grinding too.
- The model is the basis of the computer simulation.
- The model is usable in the study and design of the grinding processes.
- The model is applicable to prevent the breakdown.
- The application of the model meets the requirements of the environment-protection.
- The application of the model contributes to the optimization of the producing.

## Applied research methods

The research work involved the followings: the studying of the literature, the theoretical foundation of the dissertation, the carrying out the numerical investigations, the evaluations of the results, and deduction of conclusions.

Fundamentally the scientific publications issued from the year 1941 to the year 2003, referring to the grinding have been studied, especially the mathematical descriptions of the grinding. For describing the open circuit continuous grindings, the “fully mixed” mathematical model of Whiten is widely spread (Whiten, 1974), as well as the model of Mika (Mika, 1976) for characterizing the steady state of the mill. Application of the continuous grinding equation of the open circuit continuous grinding is also proposed (Mihálykó et al., 1998). Based on the residence time distribution, further method is available for calculating the characteristics of the ground material (Keviczky et al., 1984). Use of Markov-changes is also proposed (Berthiaux, 2000).

The newly developed models for describing the closed circuit continuous grinding – similar to the well known, often used models (Whiten, 1974, Mika, 1976, Mihálykó et al., 1998) – are based on the principles of conservation and balance of the physical chemistry systems.

The theoretical basis of the dissertation is the continuous mathematical model of the closed circuit continuous grinding. The analytical solution of this model is not known – as it is established via using the theory of the second order partial differential equations. By means of the numerical analysis – namely by the numerical methods of the differential equations, as well as by the numerical differentiation and integration methods – the approximate solutions of the models were obtained. As regarding to the topic of the numerical analysis, the numerical convergence-analysis and the constrained function minimizing methods were also used.

The methods of the linear algebra were applied too: for constructing the matrix forms of the models from the sets of recursive linear equations and for the eigenvalue analysis. The theory of the retarded difference equations was used for the obtaining the appropriate forms of the models to the convergence analysis.

During the mathematical discussion of the discrete models it was established – using the means of the mathematical modelling – that the models satisfy the natural law and their behaviours are in accordance with the expectations.

The methods of the theory of programming have been applied for the planning of the algorithms and the computer programs, while programming methods have been used for the implementation of the simulation. Computer programs were developed in programming language C and using the program package Matlab. By the means of the probability and the mathematical statistics the calculations and the evaluations of the results were accomplished. The calculations of the statistical characteristics were executed as ones are usually in the mathematical statistics.

Numerical discussions of the models have been executed. Statements and conclusions have been deduced from the results of the calculations. Tables and figures have been also used to publish the results.

## References

- Berthiaux, H., Analysis of grinding processes by Markov chains, Chem. Eng. Sci., 55 (2000) 4117-4127.
- Keviczky, L., Hilger, M., Kolostori, J., Folyamatidentifikáció, folyamatirányítási kutatások a szilikátiparban 3., Szabályozástechnikai örlélmélet, SZIKKTI, Budapest, 1984.
- Mihálykó, Cs., Blickle, T., Lakatos, G. B., A simulation model for analysis and design of continuous grinding mills, Powder Technology, 97 (1998) 51-58.
- Mika, T. S., A solution to the distributed parameter model of a continuous grinding mill at steady state, Chem. Eng. Sci., 31 (1976) 257-262.
- Whiten, W. J., A matrix theory of comminution machines, Chem. Eng. Sci., 29 (1974) 31-34.

## New scientific results

The new scientific results presented in the dissertation are summarized in the following theses.

### Thesis 1.

Mathematical models have been elaborated for describing and studying of the continuous grindings carried out in ball mills.

- 1.1. Continuous mathematical model has been developed for mathematical describing of the closed circuit continuous grinding.
- 1.2. Starting from the continuous mathematical model of the closed circuit continuous grinding the discrete mathematical model of the closed circuit continuous grinding has been derived. In the resulted model the breakage and the flow of the particles are considered as mutually exclusive events. (Model I.)
- 1.3. Second discrete mathematical model of the closed circuit continuous grinding has been deduced from Model I. The breakage and the flow of the particles are considered as independent events in the new model. (Model II.)
- 1.4. It has been proved that the continuous and the discrete mathematical models of the closed circuit grinding involve the models of the open circuit continuous grinding and the batch grinding.
- 1.5. On the basis of the discrete mathematical models, computer simulation has been developed for the study of the continuous grinding processes carried out in ball mills.

[1], [4], [5], [7].

### Thesis 2.

It has been proved the non-negativity and the convergence of the solutions of the discrete model-equations. With the aid of numerical experiments it has been proved that the results yielded via the simulation meet the expectations and ones are in accordance with the expected behaviour of the mill.

- 2.1. In the case of both Model I and Model II, it has been proved that all coefficients of the set of linear equations of the closed circuit continuous grinding are non-negative as a consequence of the appropriate choice of the simulation time-step.
  - 2.2. It has been established that the calculation results of the numerical experiments are convergent as a consequence of the refining of the distributions of the particle-size and the mill-length. According to the witness of the numerical experiments, in the case of the appropriate choice of the number of the sections and the particle-size intervals, the accuracy of the calculation results is satisfactory.
  - 2.3. On the basis of the numerical experiments, it has been proved that the results of the computer simulation are in great accordance with the empirical results published in the literature.
  - 2.4. Providing the realization of certain conditions, on the basis of the numerical experiments it has been shown that the steady state of the continuous grinding process is reached at the empirical values of the functional and kinetic parameters.
- [6].

### **Thesis 3.**

The mathematical discussion of the discrete models has been executed. The most important properties of the discrete models have been proved.

- 3.1. Assuming the realization of certain conditions, it has been proved that the amounts of the materials in the sections of the mill are constant at every moment of time. As a consequence of the above statement, the amounts of the materials in the sections of the mill are constant in the steady state of the mill, supposing the realization of certain conditions.
- 3.2. It has been proved that the amounts of the materials in the sections of the mill are equal to each other at the steady state of the mill supposing that the values of the functional and kinetic parameters are constant, and some other conditions, respectively.
- 3.3. It has been shown that the amount of the fine product is equal to the amount of the fresh particles.
- 3.4. Comparing the open circuit and the closed circuit continuous grinding processes, it has been proved that the amount of the material in the mill during the closed circuit grinding process is at least as much as it is in the open circuit process, assuming the realization of the same conditions in the open and closed circuit processes.
- 3.5. In the case of the closed circuit continuous grinding, it has been proved the formulae for express that increase of the amount of the material in the mill that happened from the beginning of the process by the steady state.
- 3.6. Assuming the realization of certain conditions, it has been proved the formulae regarding to the amount of the freshly fed material in order to avoid the overloading of the mill.
- 3.7. Assuming the realization of certain conditions, it has been proved the formulae expresses how the amount of the material in the section of the mill at the steady state depends of the amount of the fresh material. It has been proved the formulae expresses how the amount of the recycled material depends of the amount of the freshly fed material and the amount of the material in the section of the mill.

[1], [5].

### **Thesis 4.**

Via numerical experiments it has been proved that the computer simulation based on the discrete models mentioned in Thesis 1. is usable in the study and design of the continuous grinding processes and in the realization of the safe operation of the mill.

- 4.1. In the case of the open and the closed circuit continuous grinding, it has been proved that the average particle-size of the ground material at the steady state is approximately directly proportionally decreases to the length of the mill, while the distribution of the ground material at the steady state is approximately directly increases to the mill-length.

The size-reductions of the brittle and the less brittle materials are remarkable during the closed circuit continuous grinding. The length of time required to reach the steady state is much more length during the closed circuit continuous grinding. The difference is less in the cases of the brittle materials.

- 4.2. Method has been developed for establishing the optimum values of the functional parameters in order to produce the required product.
- 4.3. Regarding to the convective flow in the mill it has been established that the average particle-sizes of the ground material and the product are directly proportionally increase to convective flow, while the distributions are hardly influenced.
- 4.4. Regarding to the axial dispersion in the mill it has been established that the average particle-sizes of the ground material and the product are directly proportionally increase to the axial dispersion, the distributions are only a bit influenced. Increase of the axial dispersion leads to a little decrease of the dispersion of the product.
- 4.5. Regarding to the kinetic parameters it has been established that the average particle sizes both of the ground material and the product increase due to the increasing the values of the parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ , and due to the decreasing the values of the parameters  $\Phi$  and  $K_s$ . It has been also established that the distributions decrease due to the increasing the values of the parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ , and due to the decreasing the values of the parameters  $\Phi$  and  $K_s$ .
- 4.6. It has been shown that the calculation results of that modified models which ones take into account the influence of the functional and kinetic parameters are in greatly accordance with the empirical establishments.
- 4.7. It has been shown that the discrete models developed are usable in the design and in the process control of the grinding plat.
- 4.8. Regarding to the transient state it has been shown that the statistical characteristics of the ground material change mostly at the beginning of the process. The length of time required to the reach the steady state is established reliably using the simulation.
- 4.9. On the base of numerical experiments it has been established the characteristics of the change of the amount of the material being in the mill.
- 4.10. It has been proved that the average particle-size calculated with Model I is less than calculated with Model II, while the dispersion calculated with Model I is the greater.

[2], [3], [8], [9], [10].

## Applicability of the new scientific results

The results can be applied in the modelling of the continuous grinding carried out in cylinder ball mills, as well as in the modelling of some types of air jet mills, and in another types of grinding mills after executing little modifications. These grinding devices are widely spread in the industry: in the cement and iron manufacture, in the ceramics, silicate, pharmaceutical, food industry, as well as in the preparing of the mineral matters.

The greatly agreement between the simulation and the calculated results expresses the hope that the models are applicable in the mentioned areas without any difficulties and advantages are expected from the applications.

The models are usable in the design of the grinding-classification systems. The over-grinding of the product may be avoided, and the overload of the mill may be prevented, as well as the requirements referring to the product may be profitably satisfied using the simulation.

Mathematical modelling of the grinding is important from the point of the environment-protection too, because of the well designed processes treat carefully the environment.

The results presented in the theses show that computer simulation of continuous grinding processes may advantageously contribute to the deeper understanding of grinding processes, and may play important role in the solution of problems origin from the grinding plants and the research works.

## List of publications connecting to the dissertation

### Journal papers (issued/accepted)

- [1] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Mathematical models for simulation of continuous grinding process with recirculation, *Acta Cybernetica*, 15 (2002) 529-545.
- [2] Kis, P. B., Mizonov, V. E., Mihálykó, Cs., Lakatos, G. B., Aszimtoticeszkie rezsenija raznosztnyih uravnyenijj kinetiki uzmelcsesznyija v zamknutom cikle, *Hímija i Himicseszskaja Technologija*, 47 (2004) 131-133.
- [3] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Optimizing design of the continuous grinding mill-classifier systems, *Chemical Engineering and Processing*, 44 (2005) 273-277.
- [4] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Mathematical models and computer programs for simulation of continuous grinding processes with classification, *International Journal of Mineral Processing*. (Accepted.)

### Conference papers

- [5] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Mathematical models for simulation of continuous grinding process, *Proc. The Seventh Fenno-Ugric Symposium on Software Technology, Szeged*, (2001) 155-167.
- [6] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Discrete modelling of continuous grinding mill-classifier, *Proc. 4<sup>th</sup> International Symposium on Mathematical Modelling, Bécs*, (2003) CD-ROM, 1067-1073.
- [7] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Mathematical models and computer programs for simulation of continuous grinding process with classification, *Proc. 10<sup>th</sup> European Symposium on Comminution, Heidelberg*, (2002) CD-ROM, P03, 1-13.
- [8] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Optimising design of the continuous grinding mill-classifier, *Proc. The 4<sup>th</sup> International Conference for Conveying and Handling of Particulate Solids, Budapest*, (2003) Vol. 1, 8.69-8.74.
- [9] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Comparison of two discrete models for closed mill-classifier systems, *Proc. of 16<sup>th</sup> International Congress of Chemical and Process Engineering, Prága*, (2004) CD-ROM, F7.8.

### Papers in Hungarian language

- [10] Buzáné Kis P., Mihálykó Cs., Lakatos G. B., *Proc. Recirkulációs őrlési folyamat számítógépes szimulációja, Műszaki Kémiai Napok '03 Konferencia, Veszprém*, (2003) 392-397.
- [11] Buzáné Kis P., Mihálykó Cs., Lakatos G. B., Markov-láncok alkalmazása zárt folyamatos őrlés modellezésére, *GÉP*, 6 (2004) 14-15.

### International conference presentations

- [12] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Mathematical models for simulation of continuous grinding process, *The Seventh Fenno-Ugric Symposium on Software Technology, Szeged*, 2001.
- [13] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Numerical method for solving the population balance equation of continuous grinding processes, *Numerical Methods and Computational Mechanics, Miskolc*, 2002.
- [14] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Discrete modelling of continuous grinding mill-classifier, *The 4<sup>th</sup> International Symposium on Mathematical Modelling, Bécs*, 2003.
- [15] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Optimising design of the continuous grinding mill-classifier, *The 4<sup>th</sup> International Conference for Conveying and Handling of Particulate Solids, Budapest*, 2003.
- [16] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Asymtotic behaviuor of the delay difference equations of the grinding mill-classifier system, *International Conference on Delay Differential and Difference Equations with Applications, Veszprém*, 2003.
- [17] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Comparison of two discrete models for closed mill-classifier systems, *16<sup>th</sup> International Congress of Chemical and Process Engineering, Prága*, 2004.

### Poster

- [18] Kis, P. B., Mihálykó, Cs., Lakatos, G. B., Mathematical models and computer programs for simulation of continuous grinding process with classification, *10<sup>th</sup> European Symposium on Comminution, Heidelberg*, 2002.

### **Home conference presentations**

- [19] Buzáné Kis P., Mihálykó Cs., László Z., A matematikai modellezés és számítógépes szimuláció néhány tapasztalata, Matematika-Fizika-Informatika Főiskolai Oktatók Országos Konferenciája, Zalaegerszeg, 2001.
- [20] Buzáné Kis P., Mihálykó Cs., Lakatos G. B., Recirkulációs őrlési folyamat számítógépes szimulációja, Műszaki Kémiai Napok '03, Veszprém, 2003.