***Zsolt Tibor Kosztyán***

# OPTIMAL RESOURCE ALLOCATION

PhD Thesis

Supervisor: Dr. Andrea Bencsik

University of Veszprém

Management Doctoral School

Veszprém

**2005**

## Introduction

During the execution of a project (investment, innovation etc.), three important parameters must be kept in mind: we have to execute the project as soon as possible, with minimal total cost and not to exceed resource (manpower, materials, engines etc.) availabilities.

Why does it important to execute the project as soon as possible with minimal total cost? If more than one company compete for the execution of an investment project, usually the chance of winning the tender will be higher if a company can execute the project with minimal total project time (TPT) and minimal total project cost. This problem could already be handled in the 60s and 70s with network planning (CPM, MPM, PERT etc.), scheduling (Gantt Diagrams, LOB etc.) and other related cost-minimizing (CPM/COST, MPM/COST, PERT/COST etc.) techniques. The most difficult problem was to handle the resources. During the execution of a project we must keep in view the resources, because these resources are usually straitened. There are well-defined number of labours, engines and so on.

If we would like to execute the project with minimal TPT and minimal total project cost and optimal use of the resources (manpower, materials, engines etc.) the problem becomes easily so hard to solve (already at 5000-10000 activities) that computers available today cannot find the solution within a reasonable time. The real problem is more complicated, because before the execution of the project we can only estimate the duration time, (variable) cost and resource need of activities.

## Summary

Network planning and resource allocation techniques can be used in different fields of logistics and project management. These techniques are widely used in production, planning, distribution, installation, resource management, financial planning and project management to solve different kinds of problems.

With the help of the evolution of computer science such hard problems can be solved, where to find the optimal solution appeared unimaginable earlier. This kind of complexity (NP-hard) is the main problem of optimal resource allocation. In most cases investment companies try to find a feasible solution of the problem, so that not to exceed the constraints of resources and the TPT. This kind of feasible solution can be found relatively easily (in case of few activities) manually or (in case of many activities) by any kind of Project Management Software (for instance Microsoft Project, CA-SuperProject, Primavera etc.). However, there are some problems that emerge in case of this so-called feasible solution of the resource allocation problem. In the real life (especially in the case of the building and investment projects) it can be important to use of resources as smooth as possible, or to execute activities as soon as possible. These requirements can be defined as a target function. The ***optimal solution of a resource allocation problem with a given target function*** is a kind of feasible solution of this problem, where the requirements are mostly satisfied (otherwise the target function is minimal).

The difficulty in this problem is that three different point of view has to be considered during the execution of a (investment, innovation etc.) project: the project must be executed with minimal total project time and minimal total project cost, and not to exceed the resource availabilities. Moreover these parameters can only be estimated.

The previously applied methods can be separated into two different groups. There are algorithmic methods (for example optimal resource balancing methods), and there are heuristic methods (typical examples are the different kinds of allocation methods). The heuristic methods are usually faster then the algorithmic ones, but do not guarantee the optimal solution of resource allocation problems and in some cases different kind of heuristic methods find different feasible solutions.

In the (PhD) dissertation new methods are introduced which can be great possibilities for the management. With these methods a minimal total project cost and minimal total project cost with optimal resource allocation of resource-constrained allocation problem of any kind of project can be determined according to the uncertainties of the parameters (duration times, costs, resource needs). The methods introduced here can be used in every cases when the target function executes the activities as soon as possible with minimal cost and optimal use of resources. Especially these methods can be used in the field of project management, resource planning and in the methodology of small-scale series production management.

When I have created my dissertation my objective was to find methods to solve these problems (determined above). My methods can be applied for resource-constraint resource allocation problems to determine the optimal resource allocation for renewable (like works, engines etc.) resources. These methods are also algorithmic methods. Thus these methods guarantee to find the optimal solution when a target function is given. These methods start from a feasible solution which can be determined with any kind of heuristic method. Target function can be (for instance) the execution of the activities as soon as / as late as possible.

These methods improve the feasible solution in every step according to the given target function, until the optimal solution is found and the intermediate steps are all feasible.

In case the resources and the duration times of activities are changing at projects in progress, a new resource allocation for the running activities and for those still not started can be determined with these proposed methods.

With the help of these methods one can determine the total project time, total project cost and optimal resource allocation for any kind of project, when the parameters of this project (duration times, cost and resource need(s)) can only be estimated.

This method is capable of using different resources and can be applied in parallel projects and also can handle the not-used resources.

In my work I developed a method, which can be used efficiently in the field of project management. The goal was to develop efficient methods for project planning for project managers. The proposed methods can be applied in case of smaller (50-150 activity, few resources), and for greater (5000-50000 activities 10-20 kind of resources) projects If there is more information about the parameters of the project, the planning of the project can be more accurate and more reliable. In contradistinction to the previously published algorithms, the introduced methods can determine an optimal resource allocation with minimal total cost from a feasible solution when a target function is given. The duration time, demand of resources and cost of activities can either be deterministic or stochastic variables.

## Finding Optimal Resource Allocation from any Feasible Solution

The developed deterministic resource allocation method schedules the activities in the alternative paths of an admissible resource allocation. A feasible resource allocation can be determined for instance by ERALL-method, (which developed in Hungary), the sequential or parallel resource allocation. Feasible solution can be determined with any kind of Project Management Software (PMS) (for instance Microsoft Project). This method schedules activities to satisfy a given target function, which can be defined before the optimization process (for instance start the activities as soon as possible, balanced resource allocation and so on).

Before the detailed representation of these methods some definition (used in scheduling) must be introduced.

Some activities can be executed in parallel, but other ones must be executed in a sequential fashion. When we define successor activities of an activity, we properly define the successor relation between two or more activities.

If any kind of activities are determined for executing in the project, and successor (or predecessor) relations are determined, an exact logical network can be sketched. (This network must be directed acyclic graph with one start and one final node). This logical network shows us the logical relations (successor or predecessor relations) between two activities.

After that the (expected) ***duration time*** of the activities must be determined or must be estimated. Thereafter the ***total project time*** (TPT) can be determined with any well known network planning technique (for instance CPM, PERT, MPM).

With the help of these techniques the earliest/latest start and finish time of the activities can be calculated. Those activities, which cannot be delayed (the ***total flow*** (TF) of the activity = ***latest start time*** (LST) of the activity – ***earliest start time*** (EST) of the activity = ***latest finish time*** (LFT) of the activity – ***earliest finish time*** (EFT) of the activity = 0), are defined as ***critical activities***. The one or more series of the critical activities from the start point to the end (final) point is/are the ***critical path(s)***. This/these path(s) will be the longest path in the network.

After solving the scheduling problem, the number and quantity of the required resources of activities can be determined or can be estimated (for instance how many labours, how many materials can be used, and how many machines needed for executing a project). If the resource availabilities are given then a feasible solution must be determined. In this case such a feasible schedule is given, where the total project time is not increasing (if it is possible), but the resources are not overloaded at anywhere.

In the final step the use of resources must be determined, for this reason the target function must be determined.

If there is a feasible solution, then one can find the best solution for a given target function with a Branch and Bound method in finite step. This best solution hereinafter called ***optimal resource allocation for a given target function***.

## Optimal Resource Allocation when the availabilities of resources are constant by sections

In the real life the availabilities of resources are often not time-invariant (not constant) functions. (For example in the weekend the number of manpower is lower than in the weekdays.)

In that case when the availabilities of resources are time-variant functions (for instance constant by sections functions, and the number of cuts are finite), I show in my dissertation, that the problem can be traced back to resource allocation problem with constant (time-invariant) function of resource availabilities.

## Optimal Resource Allocation in On-line scheduling

It often turns out that one has to encroach into a project that is already in progress. It could be because of many reasons. It could happen that there are some problems in the execution of the projects. For example an activity needs more duration time or resource(s) than the planned parameters. It could be possible that the availabilities of resources are changing.

There are some activities which are interruptible, but there are a lot of non interruptible activities too. However, it is valid for interruptible activities too that they are uninterruptible for a certain time period (for example because of the technology), and a time period can be also defined after that the interrupted activity must be continued by all means. If the interruptions are allowed than the serial or parallel resource allocation can be used to find a feasible resource allocation.

If the optimisation algorithm starts from this feasible solution provided that the parts of the interrupted activities are handled as different kind of activities (keeping in view the maximal interruption times) an optimal solution ca be determined for this problem..

## Cost, time and resource simultaneous optimisation

In this case three criteria (minimal TPT, minimal TC, and optimal resource allocation) are equally important requirements. Among all the methods that were detailed earlier, the following algorithm is the most applicable in the real life. Let us assume that all of the necessary activities, variable cost, duration time and demands of resources of the activities have already been determined (or estimated). It is known that in case that the duration time of the activities is decreasing so increases the variable cost and the resource requirement. In this case a maximal parallelised optimal resource allocation has to be determined with minimal TPT and minimal amount of increasing total variable cost.

In many cases these three criteria cannot be satisfied simultaneously without any compromise.

In the calculating of this method first the normal and the minimal total project time (TPT), and a TPT with minimal total cost (TC) have to be determined. After that a feasible resource allocation has to be determined when the total cost is minimal. If there is a non-critical resource allocation, then there is an optimal resource allocation with minimal total cost. If there is no non-critical feasible resource allocation then the nearest lowest TPT can be chosen, and this way a non-critical resource allocation can be found.

If there is no any non-critical feasible resource allocation between the normal and minimal TPT then only the earlier detailed methods can be used.

### Finding alternative solutions

In some cases the offered remuneration is lower than the estimated total costs. There are three possibilities to handle this problem. We can sign away the implementation, or we can accept it, knowing that we will loose money, but can get it back later in another project. In this case we have to determine the optimal resource allocation with minimal total cost. In the scheduling phase we can use some cost minimizing method. After the scheduling phase we can determine the lower and the upper bound of the start time of the activities. After scheduling we can determine a feasible solution and then the optimal resource allocation. The third possibility is to accomplish the project and find alternative implementation of activities that requires lower variable costs. During the search for alternative implementation of activities the most important aspect should be the quality, and the decrease of costs is only the second one. If the total cost is lower than the offered remuneration we have to find the optimal resource allocation for the problem. Sometimes we cannot find alternative implementations, e.g. when a minimal quality level is given. In this case we can decide to either refuse or accept the implementation based on the extent of possible loss of money.

### Multi resource management

In most of the projects usually not only one resource has to be taken into consideration, but generally more (for example manpower, use of materials, machines etc.). In this case resources must be handled simultaneously. In this way resource availabilities can be determined for each used resources. Then resource graphs must be determined for every resource. These kinds of resources can be optimised only simultaneously, because the resulted schedules must be agreed with each other. Every task must start, finish, be interrupted and continue at the same time for all resources.

### Multi project management

More than one investment project or production can be executed parallel at a company. If these projects or productions require the same kind of resources then it is worth to redistribute some resources among these projects in order to use resources more effectively. Resources between two projects can be redistributed only if these resources are of the same kind, and the redistribution is possible (there is no spatial, geographical, technological or any kind of other obstacle) and the redistribution is allowed by the company.

If the conditions above are satisfied, the networks and the resource graphs can be joined into one complex network and one complex resource graph. The total used slack times in the feasible resource allocation cannot be lower than the earliest schedule of the activities (where these values are zero in every activity), because the successor/predecessor relations must be kept. Nevertheless the optimal resource allocation of the complex network often will be better then the optimal resource allocation of the separated networks.

If any of the conditions detailed above is not satisfied, then the optimal resource allocation can be determined only for the separated networks.

The residual resources can be variously distributed among the projects. For example if the resource is the manpower, then only so many workers have to be employed in the given time that is required by the project resources. If the resource is not manpower, then these resources can be distributed as follows: (if the resource is any kind of material or any kind of machines)

1st If the main requirement is the stability of the project progress (TPT is not allowed to increase even if the demands of resources are increased), it is practical to distribute them equally among the projects or using weight factors or priorities in the distribution.

2nd If a company choose the equal distribution for residual resources among the projects, then there are a lot of redundant transports between two projects. If the target function is the fewer number of the transport between two projects then we optimize to the minimal transport. (Assume that the demands of resources are not higher than the resource availabilities. At the optimisation the cuts of resource availability functions will be minimal)

## Optimal resource groups, resource team management

One of the new efforts of the management is to work with a team or groups. This effort has a great importance in the solutions of complex problems, preparations of decisions and also in the execution of the projects. For example there is a standard practice in order to execute some activities by a team. In many cases a lot of activities executed by subcontractors and the prime contractor only coordinates the execution of the project.

The main imperfectness of the resource allocation methods used in project management is that these cases are handled very difficultly. This method (detailed in the dissertation) can be used also in these practical cases. The goal is to determine the optimal distribution of activities within the team or subcontractors in order to cost-effectively execute these activities with minimal durations, not to overrun the resource availabilities.

This method can be started from any feasible resource allocation created by any project management software (e.g. Microsoft Project, CA-SuperProject, Primavera etc.). This method takes into concideration the workers/labours competence in the team. Therefore punctual schedule and resource allocation can be determined. These allocations can be improved in every step. If there is more than one project and/or more than one kind of resources the optimal resource allocation with minimal total cost also can be determined. And this method also can be used in multi resource or multi project management.

### Handling the uncertainty in time, cost and resource management

The duration time of activities of projects very often cannot be estimated correctly in real life. Especially in research and development programs where the duration time of activities is very slightly known and the ex ante and ex post duration times are often different.

With this algorithm an optimal resource allocation with minimal total cost for any arbitrary project could be determined. This algorithm could hopefully be widely used in project management, resource planning and in the methodology of small-scale series production management.

This method schedules the activities in the alternative paths of an admissible resource allocation, satisfying a given target function and taking into account that the duration times of the activities are probability variables with an expected value and standard deviation. According to former studies 10-12% cost can be saved if the duration times of activities are handled as probability variables instead of deterministic values, hence the uncertainty of duration times can be managed and the total project time can approximately be determined if a significance level is given. After all, the total project time is many times influenced by unanticipated events. In case when the resources and the duration time of the activities are changing in the projects in progress, a new resource allocation for the running activities and for those still not started can be determined with this method.

With this method an optimal resource allocation with minimal total project time and minimal total cost can also be determined, with minimal uncertainty of the execution and the project is maximally parallelised.

This method also can be used in multi project and/or multi resource management, and can be handled the residual resources.

## Hypothesis:

**H1.** **Different kind of practical methods in the organisations are not capable of solving any kind of cost and resource allocation problems used by the management.**

**H2.** **Algorithms used in real life cannot handle huge projects (where the number of activities are more than 10000), and there is no method to determine at least a feasible resource allocation within a reasonable time.**

**H3.** **Methods, algorithms and any kind of project management software cannot handle the uncertainty of the demanded costs and resources of any kind of activity.**

## Thesis:

**T1.** **It is proved that a best resource allocation can be determined in finite step from any kind of feasible resource allocation of a resource-constrained resource allocation problem if a target function (start as soon as / as late as possible) is given (and if the resources are renewable resources). This best**

**resource allocation will be the optimal resource allocation of this problem for a given target function. This method can be used if: [P1-P4, P7, P9, P10, C1, C2]**

- the availability function of the resource(s) is constant or constant by sections function,
- the target function is to minimize the total project time and the total cost and optimize the resource allocation,
- the goal is to handle different kind of resources simultaneously and/or distribute resources among parallel projects,
- the cases must be handled simultaneously.

**T2.** **It is proved that a changed resource constraint resource allocation problem can be optimised (in finite steps) from any kind of feasible resource allocation, if availabilities of resources and the target function are given. (If resources are renewable.) This method can be used if the followings are changing: [P1-P4, P7, P9, P10, C1, C2]**

- the availability(s) of resource(s),
- duration time of the activities,
- demand of resource(s),
- more than one of these parameters.

**T3.** **It is proved that in all cases when the expected value and standard deviation of duration time and demand of resource(s) can be determined or can be estimated, and there is a feasible resource allocation of a resource constrained resource allocation problem for a given significance level, then optimal resource allocation for a given significance level can be calculated with finite steps if a target function are determined by the management. [P5, P6, P8, P11, P13, C3-C8]**

**T4.** **It is proved that this developed method can determine an optimal resource allocation for a given significance level (if a target function is given) can be calculated from any kind of feasible resource allocation. This method can be used if the followings are changing: [P5, C3-C8]**

- the availability(s) of resource(s),
- duration time of the activities,
- demand of resource(s),
- more than one of these parameters.

**T5.** **It is proved that in all cases when a deterministic or a stochastic function can be determined between duration time and the demanded variable cost of activity(s) and if there is a feasible resource allocation (and it is assumed that the estimated variables are (strongly) stacioner ergodic variables) then it can be calculated the resource allocation program for:[P1-P4, P8, C3]**

- minimal total cost,
- minimal total project time (where the decrease of (variable) cost is minimal).

# Publications:

**[P1]** ***Kosztyán Zsolt Tibor***, Bencsik Andrea: Erőforrás-optimalizálás, egy új módszer alkalmazása a projektmenedzsmentben. Logisztikai Híradó 2001/6 5-7 oldal.

**[P2]** ***Kosztyán Zsolt Tibor***, Bencsik Andrea, Hogyor András: Egy új módszer alkalmazása idő-, erőforrás-, költségoptimálásra projekt-menedzsmentben, illetve logisztikában, 2002 Logisztikai Évkönyv 99-109 oldal.

**[P3]** ***Kosztyán Zsolt Tibor***, Bencsik Andrea, Hogyor András: Működő projektek optimális erőforrás elosztása, Műszaki Vezető, Verlag Dashöfer 2002.

**[P4]** ***Kosztyán Zsolt Tibor***, Bencsik Andrea, Hogyor András: Működő projektek optimális erőforrás elosztása, Gyakorlati Logisztikai Füzetek, Verlag Dashöfer 2002.

**[P5]** ***Kosztyán Zsolt Tibor***, Bencsik Andrea: Bizonytalan átfutási idejű projektek optimális erőforrás elosztása, Műszaki Vezető, Verlag Dashöfer 2003.

**[P6]** ***Kosztyán Zsolt Tibor***, Bencsik Andrea: Bizonytalanság kezelése különböző logisztikai feladatok esetén, Műszaki Vezető, Dashöfer 2003.

**[P7]** ***Kosztyán Zsolt Tibor***, Bencsik Andrea, Mátrai Rita: Költségcsökkentés alternatív megoldások segítségével, Műszaki Vezető, Verlag Dashöfer 2004.

**[P8]** ***Kosztyán Zsolt Tibor***, Bencsik Andrea, Mátrai Rita: Bizonytalanság kezelése kissorozatgyártás termelésirányításában, Műszaki Vezető, Verlag Dashöfer 2004.

**[P9]** Mátrai Rita, ***Kosztyán Zsolt Tibor***, Póta Szabolcs, Bencsik Andrea: Erőforrás-tervezés egy létesítményprojekt megvalósítása esetén. Esettanulmány. Műszaki Vezető, Verlag Dashöfer 2004.

**[P10]** ***Kosztyán Zsolt Tibor***, Bencsik Andrea: Erőforráscsoport, - csoportos erőforrás-tervezés, Műszaki Vezető, Verlag Dashöfer 2005.

**[P11]** ***Zs. Kosztyán,*** Sz. Póta, A. Bencsik, R. Mátrai: Distributed Deterministic and Stochastic Optimal Resource Allocation, Logistics Networks - models, methods and applications, University of Miskolc, 2004, ISBN 963 661 641 8., Eds. T. Bányai, J. Cselényi. (pp. 75-94)

***Under publication***

**[P12]** ***Zs. Kosztyán, Z. Perjés, A. Bencsik:*** Cost Reduction By Means Of Alternative Solutions, *Aston Times Office*, Aston University, Birmingham

**[P13]** ***Zs. Kosztyán,*** Z. Perjés, Sz. Póta, A. Bencsik, R. Mátrai: Deterministic and Stochastic Optimal Resource Allocation And Its Distributed Implementation, U.T.PRES, Universitea Technică din-cluj-napoca, 2005.

# Proceedings:

## National conferences:

**[C1]** ***Kosztyán Zsolt Tibor***, Bencsik Andrea: Erőforrás-optimalizálás, és alkalmazása projektmenedzsmentben és logisztikában egyedi és kissorozatgyártás

termelésirányításában, előadás: 2001 Logisztikai Konferencia, Budapest 2001 május 14
**[C2]** ***Kosztyán Zsolt Tibor***, Bencsik Andrea: Bizonytalanság kezelése erőforrás-, költség és időtervezésben, előadás: Veszprémi Egyetem, Gazdaságtudományi Önálló Intézet Doktorandusz Konferencia, 2003. március 20-21.
**[C3]** ***Kosztyán Zsolt Tibor***, Póta Szabolcs: Determinisztikus és sztochasztikus optimális erőforrás-allokáció elosztott rendszerekben. II. Gazdaságinformatikai konferencia, Győr, 2004. november 11-12, pp. 49-50. (Gyakorlati megvalósítások kategória: I. díj)

## International conferences:

**[C4]** ***Zs. Kosztyán***, A. Bencsik, R. Mátrai: Uncertainty in Optimal Resource Allocation: The 4th International Conference of PhD Students, Unversity of Miskolc, 11-17 August 2003, pp. 115-120.
**[C5]** Sz. Póta, ***Zs. Kosztyán***: Implementation of Deterministic and Stochasitc Optimal Resource Allocation in a Distributed System, microCAD 2004, Miskolc, 2004. March. 17-19., pp. 365-370.
**[C6]** ***Zs. Kosztyán***, A. Bencsik, R. Mátrai: Handling the Deterministic and Stochastic Time, Resources and Costs in Project Management and Logistics, microCAD 2004, Miskolc, 2004. March. 17-19., pp. 59-64.
**[C7]** ***Zs. Kosztyán***, A. Bencsik: Handling the Uncertainty in Resource Management, CSCS (The Fourth Conference of PhD Students in Computer Science) 2004, Szeged, 2004.July. 1-4.
**[C8]** ***Zs. Kosztyán,*** Sz. Póta: Optimal Deterministic and Stochasitc Resource Allocation in a Distributed System, CSCS (The Fourth Conference of PhD Students in Computer Science) 2004, Szeged, 2004.July. 1-4.