Review of the PhD Thesis

On the Boundedness of Discrete and Continuous Volterra Equations with Applications in Control Theory

by Essam Awwad

General description

The thesis concerns with boundedness properties of the solutions of various nonlinear Volterra integral equations with delay, delay differential equations, nonlinear Volterra difference equations with delay and delay difference equations.

The structure of the thesis is very well organized, after the introductory and the background information chapters, the four consequent chapters each describe the results for a given class of equations. Each of those chapters starts with the problem statement and some introductory part, then the main results and the proofs are presented. Some applications are given in Chapter 7 and the thesis finishes with the chapter of conclusions. Some more technical proofs are collected in the Appendix.

The nature of the mathematical results is typically as follows: sufficient conditions are given, such that the solutions of a given type of equations are bounded. Sometimes more specific estimates are given on the bounds. Another class of results states the BIBO stability of some control systems, and

also the convergence of solutions. Finding sufficient conditions for boundedness is generally one of the major first steps in the understanding of the dynamics of a given system, and it is not always a trivial task, especially for time delay systems. The main mathematical techniques used in this thesis are the properties of the fundamental solution, the variation of constants formula and various related growth estimates. The candidate has clearly mastered these techniques, and was able to skillfully use them in various contexts.

General evaluation

This review is written from a mathematical point of view, thus I did not concern with the applicability of the results in control problems of engineering, but mainly on the mathematical correctness, novelty and precision of the results. I found the thesis well written, carefully prepared and generally of good mathematical quality. The English is overall good, with some minor grammatical and semantic errors, but those do not cause any confusion. I have found only a small number of typos or minor mistakes. The mathematical results seem novel, correct and relevant, however sometimes a bit repetitive. The proofs are carefully written and sufficiently detailed. The author has published his results in five research papers, all of them in reputable international journals.

One of the core ideas is that here the control system is considered as a perturbation of a time delayed linear system (which contains the pure linear control term with delay) by the nonlinearity of the original system, as opposed to considering the control system as a perturbation of a nonlinear system by a control term. I find this 'reverse' approach interesting and it turned out to be rather useful.

The thesis book properly summarizes the problems and the results.

Comments

I have raised several questions in the preliminary review, which have been mostly adequately addressed in the revision, so I do not write about those here.

The question of the role of the matrix C has also been arisen in the preliminary review, and for me this issue is still a bit confusing. It seems that whenever some stability property is proven for $x(t)$, the similar property holds for $y(t)$ as well. The contrary is not necesserily true, one can imagine a situation when $x(t)$ is not bounded, but $y(t)$ is. In this sense, from the mathematical point of view, the boundedness of $x(t)$ is a stronger result than the boundedness of $y(t)$. Consequently, for me it would be more natural to state the boundedness theorems for $x(t)$, and then the same for $y(t)$ is a corollary. Then, the boundedness of $x(t)$ as a mathematical result itself is clear and valuable. One example for this is Theorem 4.2.1. I expect the candidate to explain and clarify in the defense what are exactly the roles and the significance of using C and y in the results.

There are several numerical simulations, which nicely illustrate the results for the delay differential and delay difference equations. However, there is none for Volterra equations. Can the candidate comment on the difficulties of developing schemes for such systems?

Summary and recommendation

Overall, in my opinion, the applicant has prepared a nice thesis. The format and the style satisfy the requirements. The results are published in good international journals, and by the amount, the novelty and the quality of the mathematical results, I recommend the thesis for public defense, and in case of a successful defense, I recommend to award the PhD degree to the applicant.

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