Reply to the referee report of Prof. Katalin Hangos

The author thank the reviewer for her thorough review. Please find below a detailed response to the comments and questions.

Note: the numbers of the references, equations, definitions and theorems are related to the Thesis.

- Could you please, summarize briefly the problems that you aimed at solving during your work?

  In this work the main goal was to find sufficient conditions implying the boundedness of solutions of large classes of Volterra integral equations and Volterra difference equations. We apply our results on Volterra equations to prove BIBO stability of nonlinear continuous time and discrete time controlled systems with time delay.

- A finite delay is introduced into the state feedback. What is the justification of this model in practice?

  It is natural to assume that there is a time needed to measure the state (or output) and to react to it in the contro mechanism. In the literature there are several papers which assume delay in the control term (see, e.g., [40], [74], [80], [81], [83], [96], [105]). The motivation of this work was to examine the role of the size of the delay in the control term. Our results show that small delay in the contol term will not destroy the BIBO stability of the controlled system.

- $dim\, x = dim\, u = d$. What is the justification of this model in practice?

  The assumption $dim\, x = dim\, u = d$ simplifies the presentation significantly. This is certainly a restrictive assumption in many applications, but there are several papers in the literature which investigates the state feedback case (see e.g. [41], [80], [81], [83], [96]). In the dissertation in Theorem 4.2.2 we formulate a sufficient condition for the BIBO stability of the controlled system in the case when an output feedback is used and the nonlinearity is sub-linear. Note that this result gives quite restrictive conditions on the feedback gain and the output matrix. Similar result can be formulated for the linear and for the super-linear case too.

- What is the relationship Definition 4.1.1 and the L2 gain of a nonlinear system?

  It is known that finite L_2-gain implies BIBO stability of a linear systems (see, e.g., the book [59]), and in some cases they are equivalent. But for nonlinear systems I do not now a result for this relation.

- What would happen, if D were not diagonal, but a full positive definite matrix?

  Yes, the assumption that D is diagonal can be relaxed. Consider equation (4.1.5) with a positive definite matrix D. A result from [60] yields that the trivial solution of (4.1.5) is exponentially stable if the eigenvalues $\lambda_1, \dots, \lambda_d$ of D satisfy inequality $0 < \lambda_j \tau < \frac{\pi}{2}$ for $j = 1, \dots, d$. If we assume that D is diagonally dominant then the sufficient condition for the exponential stability of (4.1.5) is explicit, can be given in terms of the elements of D and τ (see the second inequality of (4.2.1)).

- Please, relate your BIBO stability definition 6.1.1 to the usual one common in systems and control theory.

  Clearly, our Definition 2.1.4 in continuous case (Definition 6.1.1 in discrete case) implies Definition 2.1.1. But for nonlinear systems the opposite implication does not hold. For example consider the scalar ODE

  $$\dot{x}(t) = -\sqrt{x(t)} + u(t).$$

  If $u(t) = u$ is constant, then its equilibrium is $x = u^2$. It is easy to argue that for all bounded $u(t)$ the solutions are bounded, so Definition 2.1.1 holds. But our Definition 2.1.4 does not hold, since the equilibrium solution $x = u^2$ has no linear estimate of the the form

  $$|x(t)| \le \theta_1 |u| + \theta_2.$$

  For linear systems in some cases the two definitions are equivalent.

- What are the practical consequence of the stability analysis (how to choose the parameters α and τ)?

  First note that for $\alpha < 0.5$ the trivial solution of (7.1.5) is asymptotically stable regardless of the size of the delay. For a fixed $0.5 < \alpha < 1$ the delay τ should be selected so that

  $$\tau < \frac{\arccos \frac{3\alpha - 2}{\alpha}}{\sqrt{\alpha^2 - (3\alpha - 2)^2}}.$$

  Therefore for small enough delay the trivial solution is always asymptotically stable.

Veszprém, June 19, 2013.

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