Review on the PhD Thesis titled

PERSISTENCE AND PERMANENCE OF DELAY DIFFERENTIAL EQUATIONS IN BIOMATHEMATICS

by

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The dissertation deals with the boundedness of the positive solutions of differential equations with delayed arguments. This analysis is important for modelling permanent populations, i.e., where the population varies between two positive numbers (see Defs. 2.1.3 and 2.1.4).

The structure of the Thesis is as follows. The list of notations is given in Chapter 1. Chapter 2 overviews some basic background, known results and notions related to the dissertations. Chapter 3 deals with the persistence and the uniform permanence of the positive solutions of the general nonlinear scalar delay differential equation (3.1.1). In Chapter 4, the author gives sufficient conditions for the existence and uniqueness of the positive solutions of the general nonlinear system of algebraic equations given by (4.1.10). In Chapter 5, the author gives sufficient conditions for the uniform permanence of the positive solutions of (5.1.10). Chapter 6 summarizes the new results of the thesis. The theses are decomposed into subpoints. Thesis 1 consists of 7 points, Theses 2 and 3 consist of 2-2 points. Proofs and longer derivations are given in the appendix.

Chapter 2 provides a useful overview on the main definitions and concepts related the scope of the dissertation. Main concepts are discussed using the example of the logistic equation. An interesting point (for the reviewer) is that this equation can explain the behavior of Daphnia populations. Finally, a numerical approximation method is presented by employing piecewise constant arguments in the right hand side of the equation. This method is used for the numerical examples in the thesis.

Chapter 3 deals with a nonlinear scalar population model governed by (3.1.1). Six different specific conditions were defined and Theorems and Lemmas were stated related to the boundedness of the solution. The main result is composed by Theorem 3.2.4 and its applications are given by Corollaries 3.3.1- Corollaries 3.3.10 in Section 3.3. The last section of the chapter presents numerical examples. The core of Thesis 1 is related to the results in this Chapter. Thesis 1.1 is given by Lemma 3.2.3, Thesis 1.2 is given by Theorem 3.2.4, Thesis 1.3 is given by Corollaries 3.3.1, 3.3.3, 3.3.4, 3.3.6, 3.3.7, 3.3.8, 3.3.9, 3.3.10, Thesis 1.4 is given by Corollary 3.2.5 and Corollary 3.3.5 for a special case.

In Chapter 4, nonlinear algebraic equations are analyzed. These equations show up as steady-state equations of dynamical systems. The main result is given by Theorem 4.2.1 in Section 4.3, which states that equation (3.2.1) has a positive solution if conditions A and B hold and it has a unique positive solution if further conditions C and D are also satisfied. This Theorem gives Thesis 3.1. Section 4.3 presents the applications of the Theorem for one and two-dimensional systems. These give the core of Thesis 3.2.

Chapter 5 studies the boundedness of positive solutions of the nonlinear delay differential equation (5.1.10). As a motivation, the modelling of the bidirectional associative memory

network, secretion of the hormone testosterone and the n-dimensional Nicholson's blow flies systems is mentioned. The main results in the view of a list of conditions are given by Lemma 5.2.1 (positive solutions), Lemma 5.2.2 (bounded solutions) and Lemma 5.2.3 (uniqueness and bounds of positive solutions). These lemmas imply Theorem 5.2.4, which gives the bounded unique positive solutions of the system. The chapter gives the core of the rest of the theses. Thesis 1.5 is given by Theorem 5.2.4, Thesis 1.6 is given by Corollaries 5.3.2, 5.4.1, 5.4.2 and Thesis 1.7 is given by Corollary 5.3.1.

The PhD dissertation presents new results related to the boundedness and uniqueness of the solutions of delay differential equations, which are often used in different biological and population models. The results are given step by step in form of lemmas and theorems. The reviewer believes that the results are new and novel and they were elaborated by the applicant.

Overall, the dissertation is a nicely written document, where the applicant listed her results in a clear way. The style and the language of the dissertation is excellent, the structure follows the logic of the results.

All my comments in the previous review were addressed properly, I have no more comments or recommendations. I accept the theses as new results, therefore

I recommend the public defend

of the dissertation.

Budapest, March 29. 2017

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